# Linear Algebra

MA 631-2E, Fall 2024

**Instructor:** Dr. Y. Zeng, UH 4012

**Time & Location:** TR, 2PM – 3:15PM, UH 4002

**Office Hours:** Tuesdays and Thursdays 3:30PM–4:30PM (or by appointment)

**Text:** A set of class notes (evolved from courses taught by several faculty members in the department) will be provided. These notes contain all definitions, theorems, and examples, but no proofs (which will be presented in detail in class).

**References:**

- S. H. Friedberg, A. J. Insel & L. E. Spence, *Linear Algebra*, Pearson, 5th Ed.
- P. Lax, *Linear Algebra and Its Applications*, Wiley, 2nd Ed.
- K. Jänich, *Linear Algebra*, Springer.
- S. Axler, *Linear Algebra Done Right*, Springer.

**Course contents:** Vector spaces; linear transformations and matrices; determinants; systems of linear equations and Gaussian elimination; eigenvalues, eigenvectors and diagonalization; generalized eigenvectors and Jordan decompostion; minimal polynomials, Cayley-Hamilton theorem.

**Grading Policy:**

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| Homework assignments | 40 % |
| Midterm exam (Thursday, Oct. 10, tentative) | 20 % |
| Final exam (Tuesday, December 10, 1:30 PM – 4:00 PM) | 40 % |

**Homework Assignments:** Homework will be assigned weekly on Tuesday and due the following Tuesday, unless announced otherwise. Homework will NOT be accepted late. However, the two lowest homework grades will be dropped to account for any missed assignments due to illness or any other circumstance. I am not planning on accepting any excuses except in extraordinary circumstances.

**Exams:** Midterm and Final exams will be closed-book, in-class and comprehensive.

**Preparation for Joint Program Exam:** This course covers the material for linear algebra in the Joint Program Exams. Past exams can be downloaded at

https://www.uab.edu/cas/mathematics/graduate/phd/qualifying-exams-testbank

Problems from past exams will also be used in homework assignments.

**Learning Outcomes:** By the end of the course, students will be able to do the following.

1. Understand the concept of vector space and apply it in different situations.
2. Determine if a subset is a subspace using subspace criterion.
3. Find a basis of a vector space.
4. Change coordinates related to two different bases.
5. Know properties of matrix ranks and apply them.
6. Use equivalent criteria for independent subspaces.
7. Understand the concept of linear transformations and express a linear transformation using a matrix.
8. Understand the fundamental theorem of linear algebra and apply it.
9. Use isomorphisms to related an abstract vector space to the $n$-tuple vector space and a linear transformation to a matrix.
10. Know the concepts of linear functional and dual space, together with their properties.
11. Derive properties of determinants using the definition of determinant.
12. Know cofactor expansion, express the inverse of an invertible matrix using its adjoint and determinant, and know Cramer's rule.
13. Perform LU decomposition by Gauss elimination and express the elimination process using Gauss matrices.
14. Implement Gauss elimination, together with forward and backward substitutions, to solve a system of linear equations.
15. Implement Gauss elimination with partial pivoting and use permutation matrices to describe the process.
16. Know properties of eigenvalues and eigenvectors of a linear operator, and how its diagonalizability is related to its eigenvalues and eigenvectors.
17. Understand the concepts of generalized eigenvectors and generalized eigenspaces, and find a Jordan basis to obtain the Jordan decomposition of a linear operator.
18. Understand Cayley-Hamilton Theorem, find the minimum polynomial of a linear operator, and use it to find the Jordan from.

**Shared Values Statement:** Collaboration, integrity, respect, and excellence are core values of our institution and affirm what it means to be a UAB community member. A key foundation of UAB is diversity. At UAB, everybody counts every day. UAB is committed to fostering a respectful, accessible and open campus environment. We value every member of our campus and the richly different perspectives, characteristics and life experiences that

contribute to UABs unique environment. UAB values and cultivates access, engagement and opportunity in our research, learning, clinical, and work environments. Our department aims to create an open and welcoming environment and to support the success of all UAB community members.