MA 125-CT, CALCULUS I

February 7, 2017

Name (Print last name first): .....

TEST I

Show all your work! No partial credit will be given for the answer only!

PART I

Part I consists of questions. Clearly write your answer in the space provided after each question. Show all of your your work!

All problems in Part I are 7 points each

Question 1

Use the **definition** of the derivative to show that the derivative of $y = f(x) = 3x^2 + x$ is $f'(x) = 6x + 1$.

Question 2

Find the derivative of $f(x) = x^3 \sin(x)$

Answer: .....

Question 3

Find the derivative of $y = f(x) = \frac{x^2+1}{x+1}$.

Answer: .....

Question 4

Find the derivative of $y = f(x) = \sqrt{x}(3x + x^2)$.

Answer: .....

Question 5

Find the equation of the tangent line to the graph of $y = f(x) = 2\tan(x)$ at the point $a = 0$.

Answer: .....

Question 6

Show that the equation $x\cos(x) + 1 = 0$ has at least one solution on the interval $[0, \pi]$. Hint: use Intermediate Value Theorem.

Answer: .....

Question 7

Answer: .....

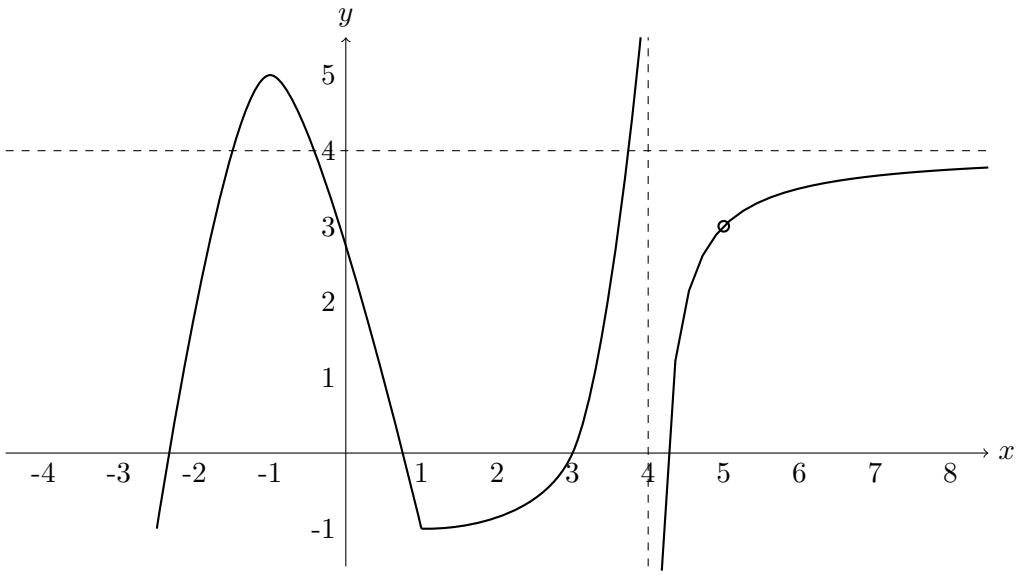
PART II

Part II consists of 5 problems. You must show correct reasons to get full credit. Displaying only the final answer (even if correct) without the relevant steps will not get full credit.

Problem 1 (10 points)

Suppose that $S(t) = t^3 - 4t^2 + t$ is the position of a particle at time t (in seconds) on a line. Find:
(a) the velocity at time t,
(b) the displacement from $t = 0$ to $t = 2$,
(c) the displacement from $t = 2$ to $t = 3$.
Recall that the displacement could be positive or negative depending on the direction of movement.

Problem 2 (12 points)



Given the graph of the function $y = f(x)$ above, find:

1. $\lim_{x \to -1^-} f(x) =$
2. $\lim_{x \to -1^+} f(x) =$
3. $\lim_{x \to -1} f(x) =$
4. $\lim_{x \to 4^-} f(x) =$
5. $\lim_{x \to 4^+} f(x) =$
6. $\lim_{x \to 4} f(x) =$
7. $\lim_{x \to 5^-} f(x) =$
8. $\lim_{x \to 5^+} f(x) =$
9. $\lim_{x \to 5} f(x) =$
10. $\lim_{x \to \infty} f(x) =$
11. State all intervals on which $f(x)$ is continuous.
12. State all intervals where $f(x)$ is differentiable.

Problem 3 (10 points)

Find all points on the graph of $f(x) = 2x^3 - 9x^2$ where the tangent line is parallel to the line $y = -12x$.

Problem 4 (10 points)

Define a function

$$f(x) = \begin{cases} \frac{x^2-x}{x-1}, & x < 1, \\ kx, & x \geq 1, \end{cases}$$

a) (8 points) Find a value of k so that $f(x)$ is a continuous function for all real numbers x.

b) (2 points) For the value of k you found, is $f(x)$ differentiable at $x = 1$? Briefly explain.

Problem 5 (10 points)

Evaluate the following limits. Like always, justify your answers.

1. $\lim_{x\to\infty} \sqrt{x+3} + \sqrt{x}$

2. $\lim_{x\to\infty} \sqrt{x+3} - \sqrt{x}$

Scratch paper