MA 125 CT, CALCULUS I

February 8, 2016

Name (Print last name first): .....

TEST I

Show all your work! No partial credit will be given for the answer only!

PART I

Part I consists of questions. Clearly write your answer in the space provided after each question. Show all of your your work!

All problems in Part I are 7 points each

Evaluate the following limits.

Question 1

Use the **definition** of the derivative to show that the derivative of $y = f(x) = 5x + 3$ is $f'(x) = 5$.

Question 2

Find the derivative of $f(x) = x^2 \sin(x)$

Answer: .....

Question 3

Find the derivative of $y = f(x) = \frac{1-x^3}{1+x^3}$.

Answer: .....

Question 4

Find the derivative of $y = f(x) = \sqrt[5]{x}(x^3 + x^5)$.

Answer: .....

Question 5

Find the equation of the tangent line to the graph of $y = f(x) = \tan(x)$ at the point $a = \pi/4$.

Answer: .....

Question 6

Evaluate the limit $\lim_{\mathbf{x}\to\infty} \dfrac{x^2 + x}{x^5}$

Answer: .....

Question 7

Evaluate the limit $\lim_{x\to 5} \dfrac{x^2-2x-15}{x^2-25}$

Answer: .....

PART II

Part II consists of 4 problems. You must show correct reasons to get full credit. Displaying only the final answer (even if correct) without the relevant steps will not get full credit.

Problem 1 (10 points)

Suppose that $S(t) = t^5 - 5t$ m is the position of a particle at time t (in seconds) on a line. Find:
(a) the velocity at time t.
(b) The displacement from $t = 0$ to $t = 2$.
(c) Is the velocity increasing or decreasing when $t = -1$?

Problem 2 (10 points)

Given the graph of the function $y = f(x)$ below find:

1. $\lim_{x \to -1^-} f(x) =$
2. $\lim_{x \to -1^+} f(x) =$
3. $\lim_{x \to -1} f(x) =$
4. $\lim_{x \to 2^-} f(x) =$
5. $\lim_{x \to 2^+} f(x) =$
6. $\lim_{x \to 2} f(x) =$
7. $\lim_{x \to \infty} f(x) =$
8. $\lim_{x \to -\infty} f(x) =$
9. State all intervals on which $f(x)$ is continuous.
10. State all intervals where $f(x)$ is differentiable.

Problem 3 (10 points)

Find all points on the graph of $f(x) = 2x^3 - x$ where the tangent line is parallel to the line $y = 149x + 7$.

Problem 4 (11 points)

Suppose $P(x) = \frac{x}{x^2+1}$ (for $x \geq 0$) describes the profit (in 100's of dollars) from selling x items (in thousands so that x=3 means 3000 items).

1. Given the definition of $P(x)$, what is the meaning of $P(2)$?

2. Compute $P'(2)$ and state what it means.

3. Considering your answer in (b), is your profit increased when you sell slightly more than 2000 items?

Problem 5 (10 points)

Evaluate the following limits. Like always, justify your answers.

1. $\lim_{x\to\infty} \sqrt{x} + \sqrt{x+1}$

2. $\lim_{x\to 0} \dfrac{\tan(x)}{x}$

Scratch paper