MA 126 - 8C CALCULUS II

April 14, 2015

Name (Print last name first): .....

Student Signature: .....

TEST IV

Closed book - Calculators and One Index Card are allowed!

PART I

Part I consists of 8 questions. Clearly write your answer (only) in the space provided after each question.
Show your work to justify your answers. Very limited partial credit or none at all for this part of the test!

Each question is worth 6 points.

Question 1

Find an equation of the sphere with center $(1, 0, 1)$ that passes through the point $(4, 0, 5)$.

Answer: .....

Question 2

Find the angle between the vectors $\mathbf{u} = \langle 0, 1, 0 \rangle$ and $\mathbf{v} = \langle 0, 1, -\sqrt{3} \rangle$. (Write your angle in degrees or radians or leave it in terms of $\cos^{-1} :=$ arccos!)

Answer: .....

Question 3

If $\mathbf{a} = \langle -2, -1, 3\rangle$ and $\mathbf{b} = \langle 2, 1, 3\rangle$, find the component (i.e., scalar projection) of $\mathbf{b}$ along $\mathbf{a}$; that is, find $\text{comp}_{\mathbf{a}}\mathbf{b}$.

Answer: .....

Question 4

A constant force with vector representation $\mathbf{F} = 6\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$ moves an object along a straight line from the point $P(1, 1, 2)$ to the point $Q(3, 1, 3)$. Find the work done if the distance is measured in meters and the magnitude of the force is measured in newtons.

Answer: .....

Question 5

Find the parametric equations of the line that is perpendicular to the plane $2x - 3y - 7z = 1$ and passes through the point $(1, 1, -2)$.

Answer: .....

Question 6

Find an equation of the plane containing both the point $P(1,1,1)$ and the vectors $\mathbf{u} = \langle 0,1,1\rangle$ and $\mathbf{v} = \langle 1,1,0\rangle$.

Answer: .....

Question 7

Find the distance from the point $P(1,2,1)$ to the plane $x + 2y + 2z = 1$.

Answer: .....

Question 8

Find the point of intersection of the plane $x+y-z = -1$ and the line given by the parametric equations

$$\ell_1 := \begin{cases} x = 1 + t \\ y = 2 - t \\ z = 3 + t \end{cases}$$

Answer: .....

PART II

Each problem is worth 13 points.

Part II consists of 4 problems. You must show your work on this part of the test to get full credit. Displaying only the final answer (even if correct) without the relevant steps will not get full credit - no credit for unsubstantiated answers!

Problem 1

This problem has two separate questions (a) and (b). Answer each question.

(a) Find the area of the triangle with vertices $P(0,0,1)$, $Q(0,-2,0)$ and $R(3,0,0)$.

(b) Find the volume of the box generated by the vectors $\mathbf{a} = \mathbf{i} - \mathbf{j} + \mathbf{k}$, $\mathbf{b} = \mathbf{i} + \mathbf{j} - \mathbf{k}$ and $\mathbf{c} = -\mathbf{i} + \mathbf{j} + \mathbf{k}$.

Problem 2

Consider the two lines given by the parametric equations

$$\ell_1 = \begin{cases} x = -1 - t \\ y = 1 + 2t \\ z = 2 + t \end{cases} \qquad \text{and} \qquad \ell_2 = \begin{cases} x = -1 + s \\ y = 2 - 3s \\ z = -1 - 2s \end{cases}$$

(a) Determine whether they are parallel.

(b) Determine whether they intersect. If they do intersect, find the point of intersection.

(c) Determine whether they are skew. If they are skew, find the distance between them.

Problem 3

Find the parametric equations of the line of intersection of the planes

$$-x + y - z = 2 \quad \text{and} \quad 2x - y + 3z = 0.$$

Problem 4

A particle is traveling along the space-curve

$$\mathbf{r}(t) = \langle 4\cos(t),\ 4\sin(t),\ 3t \rangle$$

when the time t is such that $-\infty < t < \infty$.

(i) Determine the velocity-vector of the particle at the time $t = 0$.

(ii) Find the unit tangent vector to this space-curve at the point where $t = 0$.

(iii) Find the parametric equations of the tangent line to this curve at the point where $t = 0$.

(iv) Find the arc length of this curve (i.e., the distance traveled by the particle) for the period $0 \leq t \leq \pi$.

SCRATCH PAPER

(Scratch paper will not be graded!)

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