MA 126 - 8C CALCULUS II

March 17, 2015

Name (Print last name first): ....

Student Signature: ....

TEST III

Closed book - Calculators and One Index Card are allowed!

PART I

Part I consists of 8 questions. Clearly write your answer (only) in the space provided after each question.
Show your work to justify your answers. Very limited partial credit or none at all for this part of the test!

Each question is worth 6 points.

Question 1

Determine whether the sequence $a_n = \sin\left(n\,\frac{\pi}{2}\right)$ is convergent or divergent. Find its numerical value if it converges!

Answer: ....

Question 2

Determine whether the sequence $\left\{2, \sqrt{2}, \sqrt{\sqrt{2}}, \sqrt{\sqrt{\sqrt{2}}}, \sqrt{\sqrt{\sqrt{\sqrt{2}}}}, \dots\right\}$ is convergent or divergent. Find its numerical value if it converges!

Answer: ....

Question 3

Determine whether the series $\sum_{n=1}^{\infty} \frac{-3}{n(n+1)}$ is convergent or divergent.

Answer: ....

Question 4

Determine whether the series $\sum_{n=2}^{\infty} \frac{3n^2}{n^3-7}$ is convergent or divergent.

Answer: ....

Question 5

Use the Integral Test to determine whether the series $\sum_{n=2}^{\infty} \frac{1}{n\,(\ln(n))^{1.2}}$ is convergent or divergent.

Answer: ....

Question 6

Determine whether the geometric series $\sum_{n=1}^{\infty}\left(\frac{2}{3}\right)^{n-1}$ is convergent or divergent. Find its sum if it onverges.

Answer: ....

Question 7

Determine whether the alternating series $\sum_{n=1}^{\infty}(-1)^n\frac{n}{2n+1}$ is divergent, absolutely convergent, or conditionally convergent. (Be specific here!)

Answer: ....

Question 8

Determine whether the alternating series $\sum_{n=1}^{\infty}(-1)^n\frac{1}{n!}$ is divergent, absolutely convergent, or conditionally convergent. (Be specific here!)

Answer: ....

PART II

Each problem is worth 13 points.

Part II consists of 4 problems. You must show your work on this part of the test to get full credit. Displaying only the final answer (even if correct) without the relevant steps will not get full credit - no credit for unsubstantiated answers!

Problem 1

Let $\{a_n\}$ be a sequence (i.e., listing) of numbers. Suppose none of them is zero, and that positive numbers alternate with negative ones so that a_1, a_3, a_5, . . . are positive and a_2, a_4, a_6, . . . are negative. Can such an alternating (oscillating) sequence have a limit? More precisely, which one of the following statements is correct? (Mark only one as your answer.)

(a) Such an alternating sequence cannot have a limit.

(b) Such an alternating sequence can have a limit, but the limit must be zero.

(c) Such an alternating sequence can have a limit, but the limit must be positive.

(d) Such an alternating sequence can have a limit, but the limit must be negative.

(e) Such an alternating sequence can have a limit, and the limit can be either positive or negative.

(Remember to mark one correct answer.) You should add a short (e.g., one line) explanation why you think your answer is correct.

<u>Problem 2</u>

Find the radius and interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{2^n n^3} (x-3)^n.$$

Be sure to check any endpoints that exist!

Problem 3

Express the function $f(x) = \dfrac{2x-3}{x^2-3x+2}$ as (the sum of a) power series by first using partial fractions.

Problem 4

Answer all the following questions.

(a) Find a series representation for the function $f(x) = \dfrac{1}{1+x^5}$.

(Hint: The geometric series might prove useful here, if need be!)

(b) Use the series in (a) to evaluate the (*indefinite*) integral

$$\int \frac{1}{1+x^5}\,dx$$

as a power series.

(c) Use the series in (b) to write out a series representation for

$$\int_0^{0.1} \frac{1}{1+x^5}\,dx$$

(Do not compute and add the terms of your series!)

(d) Find the <u>minimum</u> number of terms you need in the series in (c) to approximate $\displaystyle\int_0^{0.1} \frac{1}{1+x^5}\,dx$ with an error less than 10^{-9} (a nano-unit)? (Show your work!)

SCRATCH PAPER

(Scratch paper will not be graded!)

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