MA227, Calculus III
February 4, 2015
Dr. Li

Name and section: ______________________________

1. **Do not open this exam until you are told to do so.**
2. This exam has 10 pages including this cover. There are 7 questions, for a total of 100 points and 10 bonus points. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
3. Do not separate the pages of this exam. If they do become separated, write your name on every page and point this out to your instructor when you hand in the exam.
4. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.
5. **Organize your work,** in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive little credit.
6. Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.
7. Show an appropriate amount of work for each problem, so that graders can see not only your answer but how you obtained it.

Do not write in the table below.

Question	Bonus	Bonus Points	Score
1	10	0	
2	10	0	
3	20	0	
4	20	0	
5	20	0	
6	10	10	
7	10	0	
Total:	100	10	

1. Given two vectors $\mathbf{r}_1 =< 1, 1, 1 >$ and $\mathbf{r}_2 =< 2, -1, -1 >$.

   (a) [5 points] Find the angle between $\mathbf{r}_1$ and $\mathbf{r}_2$.

   (b) [5 points] Find $\mathbf{r}_1 \times \mathbf{r}_2$.

2. [10 points] Let $\mathbf{r}(t) =< \ln(t^2+1), t^2-t, e^{2t} >$ be a curve. Find the parametric equations of the tangent line to the curve at $t = 0$.

3. Let $P = (1, 0, 0)$, $Q = (0, 1, 0)$, and $R = (0, 0, -1)$ be three points in the sapce.

   (a) [10 points] Find the area of the triangle ΔPQR.

(b) [10 points] Find the equation of the plane containing the points P, Q, and R.

4. Let $\mathbf{r}(t) = t\ \mathbf{i} + \sin(2t)\ \mathbf{j} + \cos(2t)\ \mathbf{k}$ be a curve.

   (a) [10 points] Find the length of the curve when $0 \le t \le 2$.

   (b) [10 points] Find the curvature κ at point $t = 0$.

5. Let $\mathbf{r}(t) =< \sqrt{2}\cos t, \sin t, \sin t >$ be a curve.

   (a) [10 points] At $t = 0$, find the unit tangent vector $\mathbf{T}(0)$ for the curve.

(b) [10 points] At $t = 0$, find the unit normal vector $\mathbf{N}(0)$, and the binormal vector $\mathbf{B}(0)$ for the curve.

6. A particle moves along the curve $\mathbf{r}(t) =< e^{-t}, t\sin t, t^4 + 2t + 1 >$.

   (a) [10 points] Find its velocity, speed, acceleration at $t = 0$.

   (b) [10 points (bonus)] Find the tangential and normal components of the acceleration at $t = 0$.

7. [10 points] A particle moves with acceleration $\mathbf{a}(t) = 6t\,\mathbf{i} + 12t^2\,\mathbf{j} + e^t\,\mathbf{k}$. Find its position $\mathbf{r}(t)$ if the initial velocity and position are $\mathbf{v}(0) = \mathbf{i}$ and $\mathbf{r}(0) = 2\,\mathbf{j} - 4\,\mathbf{k}$.