MA227, Calculus III : Final Exam

April 22, 2015

Name and section: ______________________________

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1. **Do not open this exam until you are told to do so.**
2. This exam has 15 pages including this cover. There are 14 questions, for a total of 112 points. 100 (or more) points is equivalent to 100% for the exam.
3. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
4. Partial credit is awarded where appropriate. Show all working; your solution must include enough detail to justify any conclusions you reach in answering the question.

Do not write in the table below.

Question	Points	Score
1	8	
2	8	
3	8	
4	8	
5	8	
6	8	
7	8	
8	8	
9	8	
10	8	
11	8	
12	8	
13	8	
14	8	
Total:	112	

1. [8 points] Find the equation of the plane containing the points $(1,3,2)$, $(2,0,-1)$, and $(3,-1,1)$.

2. [8 points] Find the directional derivative of the function $f(x, y, z) = y^2 z - e^x z^2$ at $P(0, 2, 1)$ in the direction of vector $< 3, 0, 4 >$.

3. [8 points] Let $z = x^3 - xy^2$. Find the equation of the tangent plane at the point $(-1, 2)$.

4. [8 points] Find the local maximum, minimum, and saddle points (if any) of the function

$$f(x, y) = x^2 - xy + y^2 + 2x - y + 3.$$

5. [8 points] Find the linear approximation for the function

$$f(x,y) = e^{2x}\ln y - xy^2$$

near the point $(0,1)$.

6. [8 points] Evaluate

$$\int\int_D 1dA$$

where D is the triangular region with vertices $(0,0)$, $(1,1)$, and $(1,\frac{1}{2})$.

7. [8 points] Find the mass of the lamina that lies within the region $x^2 + y^2 \leq 1$, $y \geq 0$, if the lamina has density $\rho(x, y) = x^2 + y^2$.

8. [8 points] Find the unit tangent vector at the point on the curve $\mathbf{r}(t) =< \ln t, e^{2t-2}, t^2 >$ corresponding to $t = 1$.

9. [8 points] Find the maximum rate of change of $f(x, y) = 2\sqrt{x} - x^2y^2$ at the point $(1, 1)$. In which direction does it occur?

10. [8 points] Find the equation of the tangent plane to the surface $xe^z + yz + y = 2$ at the point $(1, 1, 0)$.

11. [8 points] Find the absolute maximum and absolute minimum of the function $f(x, y) = x^2 + y^2 - 2x + 1$ on the region $0 \leq x \leq 2$ and $-1 \leq y \leq 1$. (**Be sure to provide coordinates of the points and the values of absolute maximum and minimum.**)

12. [8 points] Switch the order of integration in the iterated integral

$$\int_0^1 \left[\int_{x^3}^{\sqrt{x}} f(x,y)dy \right] dx.$$

13. [8 points] Use spherical coordinates evaluate

$$\int\int\int_E x^2 + y^2 + z^2 dV,$$

where E is upper half unit ball $x^2 + y^2 + z^2 \leq 1$, $z \geq 0$.

14. [8 points] Calculate the integral

$$\int\int_D e^{x-y} dA,$$

where the region D is bounded by the lines $x - y = 0$, $x - y = 1$, $2x - y = 1$, and $2x - y = 2$. Use the change of variables $u = x - y$, $v = 2x - y$.