MA 125 CT, CALCULUS I

Test 3, March 19, 2015

Name (Print last name first): .....

Show all your work and justify your answer!

No partial credit will be given for the answer only!

PART I

You must simplify your answer when possible.
All problems in Part I are 7 points each.

1. If $F(x) = \int_2^x \sqrt{t^3+1}\,dt$, find the derivative $F'(x)$.

2. Use a Riemann sum with $n = 4$ terms and the right endpoint rule to approximate $\int_1^2 \sqrt{x^3+1}\,dx$. (You don't need to multiply or add the terms.)

3. Evaluate $\int x^3(x^2 + x)\,dx$

4. Evaluate $\int x^2\sqrt{x^3 + 1}\,dx$

5. Evaluate $\displaystyle\int \frac{x^3 + 1}{x^5}\,dx$

6. Find the average value of the function $f(x) = \sec^2(x)$ on the interval $[0, \pi/4]$.

7. Evaluate $\int \sin^5(x)\ \cos(x)\, dx$.

8. Evaluate $\displaystyle\int_{-2}^{2} \frac{x^5}{(x^6+1)^2}\, dx$

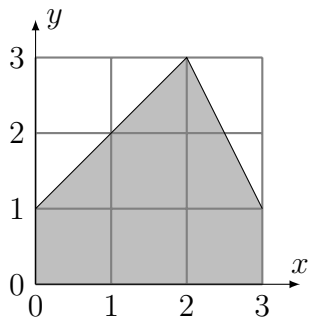
PART II

All problems in Part II are 11 points each.

1. Suppose the graph of a function $y = f(x)$ is triangular, as shown in the plot below.
   (i) Find the value of its integral: $\int_0^3 f(x)\,dx$

   (ii) Let $g(x) = \int_0^x f(t)\,dt$. Is the derivative $g'(\frac{5}{2})$ positive or negative (you must explain your answer!).

   The area of a triangle is $\frac{1}{2} \cdot \text{base} \cdot \text{height}$



2. Evaluate $\displaystyle\int \frac{x}{(1-x)^{50}}\,dx$

3. Evaluate $\int_{-3}^{3} \sqrt{9-x^2}\,dx$. (Hint: consider the graph.)

4. If a car travels with a velocity given by $v(t) = 100t^3$ find the displacement (i.e., the net distance traveled) between times $t = -1$ and $t = 2$. As a bonus, also find the *total* distance traveled over this time interval.

5. **Bonus question worth at most 5 points!** In part I, problem 2 you used a Riemann sum with $n = 4$ terms and the right endpoint rule to approximate $\int_1^2 \sqrt{x^3+1}\,dx$. Is this estimate an upper or lower estimate? As always, you must justify your answer. Find another estimate (also with n=4 terms) so that the true value of the integral is in between the two estimates. What should you use as your best approximate value and what do you know about its error? (You don't need to add and multiply the terms.)