MA 125 CALCULUS I

Final Exam, April22, 2015

Name (Print last name first): .....

Show all your work, justify and simplify your answer!

No partial credit will be given for the answer only!

PART I

You must simplify your answer when possible but you don't need to compute numbers: $\mathbf{e^6}\sin(\mathbf{12/5})+\mathbf{8}$ is a fine answer.

All problems in Part I are 4 points each.

1. Use **the definition** of the derivative to show that the derivative of the function $y = f(x) = \frac{1}{x}$ is $f'(x) = -\frac{1}{x^2}$.

2. Find the derivative $f'(x)$ if $f(x) = x^3 \sin(x)$.

3. Find the derivative $f'(x)$ if $f(x) = \ln(x^4 + x^2)$.

4. Find the derivative $f'(x)$ if $f(x) = \frac{x^2+1}{x^2-1}$.

5. Find the anti-derivative $\int x^2(1+x)\,dx$.

6. Find the anti-derivative $\int xe^{x^2}dx$.

7. Find the anti-derivative $\int x^2(x^3+5)^7\,dx$.

8. Solve $\ln(2x + 1) = 3$.

9. If $F(x) = \int_2^x \cos(t^2)\ dt$, find $F'(x)$.

10. If a rocket burns fuel at a rate of $3t^2\ kg/s$, how much fuel is burned during the first 5 seconds?

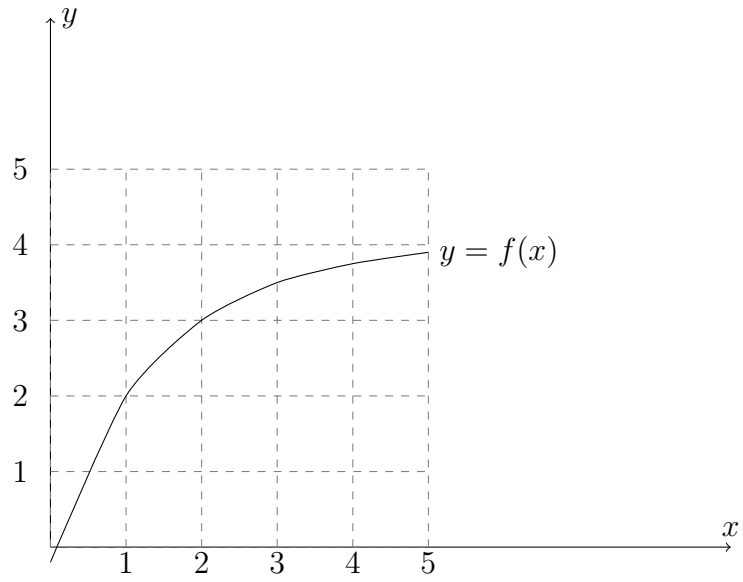
11. Approximate $\int_1^5 \frac{1}{x}\,dx$ using a Riemann sum with $n = 4$ terms and the midpoint rule. What does this number have to do with $\ln(5)$?

12. The velocity of a particle is given by $v(t) = t^3 + 1$ (m/s) .

    (a) Find the acceleration $a(1)$ of the particle,

    (b) How far does the particle travel in the first 2 seconds?

13. Given the graph of the function $f(x)$ below answer the following questions.



(a) Is $f(x)$ one-to-one? Explain!!

(b) Use the graph to approximate $f^{-1}(2)$.

(c) Use the graph to approximate $(f^{-1})'(2)$.

PART II

1. **9 points.** Find all local/absloute maxima/minima of the function $f(x) = (x+1)^5(1-x)^7$ on the real line $(-\infty, \infty)$.

2. **9 points.** Show that the equation

$$f(x) = x^3 - x + 1 = 0$$

has at least one solution.

Use Newton's method with $x_1 = -1.5$ to compute the next approximate solution x_2.

3. **12 points.** Graph the function $y = f(x) = \dfrac{x}{x^2 - 4}$. Find x and y-intercepts, horizontal and vertical asymptotes, all critical numbers, intervals of in-/de-creasing, local/absolute max/min

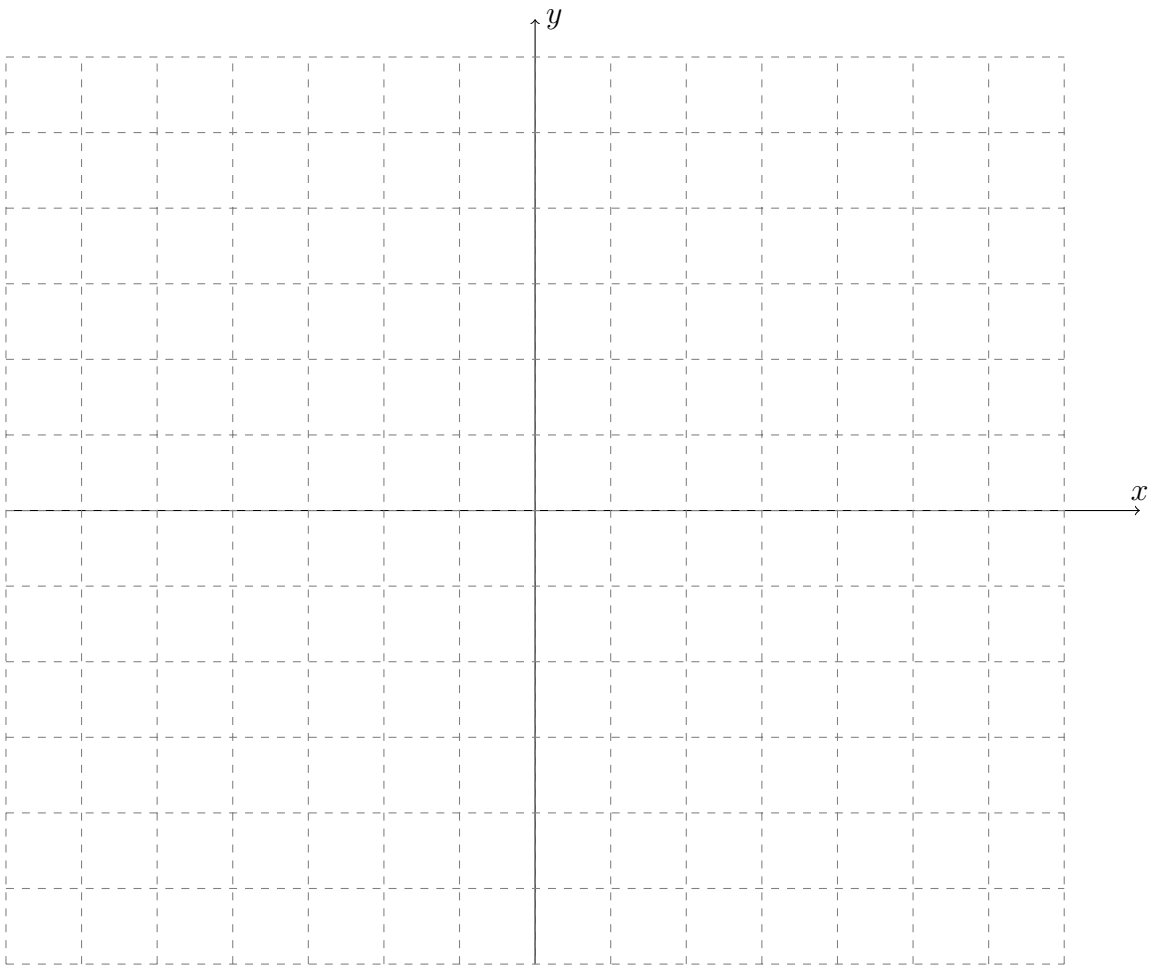
   Draw your graph on the next page. Find:

   (a) x and y intercepts,

   (b) vertical and horizontal asymptotes,

   (c) critical numbers,

   (d) intervals where $f(x)$ is in/de-creasing, and local max/min.


y
x

4. **9 points.** Find the average value of the function $y = f(x) = x\sin(x^2)$ on the interval $[0, \sqrt{\pi}]$.

5. **9 points.** A farmer wants to fence a piece of land along the side of a river. If the bank of the river is straight and he has 100 m of fence available, find the dimensions of a rectangular plot of maximal area. [Note that no fence is needed along the river.]

Scratch paper