EGR 265, Math Tools for Engineering Problem Solving
April 9, 2014, 50 minutes

TEST III

Name: .....

Problem 1	
Problem 2	
Problem 3	
Problem 4	
Problem 5	
Problem 6	
Problem 7	
Problem 8*	
Total	

Problem 1 (9+9 points)

(a) Let $g(x, y, z) = x^2y^2z^2$. Find its third order partial derivative g_{xyz}.

(b) For the function $f(x, y) = (x + y)\sin(2x)$ find f_x, f_y and f_{yy}.

Problem 2 (9+9 points)

(a) For the function $h(x, y) = (2x + 2)\sqrt{y}$ find a unit vector in the direction of steepest ascent at the point $P(1, 4)$. Also, find the rate of ascent in this direction.

(b) Find the directional derivative of $h(x, y)$ at $P(1, 4)$ in the direction of the vector $\mathbf{v} = -3\mathbf{i} + 4\mathbf{j}$.

Problem 3 (12+6 points)

(a) Find an equation for the tangent plane to the graph of $z = (x + y)^5$ at the point $(1, 0, 0)$.

(b) Also, find parametric equations for the normal line of $z = (x + y)^5$ at $(1, 0, 0)$.

Problem 4 (12 points)

Evaluate $\int_C x\,ds$, where C is the graph of the function $y = x^2$, $0 \leq x \leq 1$.

Problem 5 (12 points)

Find the work done by the force field $F(x, y) = e^x\mathbf{i} + xy\mathbf{j}$ along the curve parameterized by $x = t^2$, $y = t^3$, $0 \leq t \leq 1$.

Problem 6 (5+5 points)

Determine for each of the following force fields if it is conservative.

(a) $F(x, y) = (3x^2 - y^2)\mathbf{i} + (5 - 2xy)\mathbf{j}$

(b) $F(x, y) = (2x - y)\mathbf{i} + (x - y^2)\mathbf{j}$

Problem 7 (12 points)

For the conservative force field $F(x, y)$ from Problem 6 find a potential function $\phi(x, y)$ and calculate the work done by the force field along the curve traced by the vector function $\mathbf{r}(t) = \sqrt{4 - t^2}\,\mathbf{i} + t^2\mathbf{j}$, $0 \leq t \leq 2$.

Problem 8* (6 points bonus)

Give an example of a conservative force field $F(x, y)$ (but not the one from Problem 6). Justify!