EGR 265, Math Tools for Engineering Problem Solving

April 23, 2014, 10:45am to 1:15pm

Name (Print last name first): ....

Student ID Number: ....

Final Exam

Problem 1	
Problem 2	
Problem 3	
Problem 4 (incl. Bonus)	
Problem 5	
Problem 6 (incl. Bonus)	
Problem 7	
Problem 8	
Problem 9	
Problem 10	
Total (out of 100 + 8 Bonus)	

Problem 1 (8 points)

Find an explicit solution of the initial value problem

$$2yy' = x, \quad y(2) = 1.$$

Problem 2 (10 points)

A liquid is heated to 180°F. It cools down according to Newton's law of cooling in a surrounding medium of temperature 80°F. The rate of cooling is $k = -1/3$.

(a) State the differential equation which governs the temperature of the medium at time t according to Newton's law of cooling.

(b) Solve this differential equation with the correct initial value (this can be done either as a separable or a linear equation).

(c) At what time has the temperature dropped to 100°F? (Logarithms do not need to be evaluated.)

Problem 3 (14 points)

Consider the second order differential equation

$$y'' + 6y' + 9y = 3x - 1. \tag{1}$$

(a) Find the general solution of the homogeneous equation corresponding to (1).

(b) Find a particular solution of the inhomogeneous equation (1).

(c) Solve the initial value problem given by (1) and initial conditions $y(0) = 0$, $y'(0) = 0$.

Problem 4 (10 points + 4 points bonus)

A mass of 4 kg stretches an undamped spring by 10 cm. For simplicity, assume that $g = 10$ m/s^2.

(a) Find the spring constant k, including its correct unit. Also find the angular frequency ω of the spring-mass system.

(b) Set up the second order differential equation which governs the motion of the spring-mass system, choosing the x-axis to be oriented downwards. Find the general solution of this equation.

(c) Find the particular solution of the equation if the mass is released from 1 meter above the equilibrium position at a downward velocity of 50 cm/s.

(d) (Bonus) A damping force proportional to β times the instantaneous velocity is added to the above spring mass system. How does β have to be chosen to achieve critical damping?

Problem 5 (10 points)

(a) Find the gradient of $f(x, y) = \ln(x^2 + y)$.

(b) Evaluate the directional derivative of $f(x, y)$ at the point with coordinates $(1, 1)$ in the direction of the vector from the point $(1, 3)$ to $(3, 6)$.

(c) Find a unit vector in the direction of steepest decrease of $f(x, y)$ at the point $(1, 1)$.

Problem 6 (8 points + 4 points bonus)

(a) Determine the equation of the tangent plane to the graph of $x^2 + 3y^2 + 2z^2 = 9$ through the point $(2, 1, 1)$.

(b) (Bonus) Are there points in 3D space at which the tangent plane to $x^2+3y^2+2z^2 = 9$ is horizontal (i.e. parallel to the xy-plane)? If yes, provide all three coordinates for each one of these points.

Problem 7 (8 points)

Find the line integral

$$\int_C x^2 y \, ds,$$

where C is the straight line segment connecting the points $(0, 1)$ and $(1, 0)$.

Problem 8 (12 points)

(a) Verify that the force field $\mathbf{F}(x, y) = (y^2 - 2xy)\mathbf{i} + (2xy - x^2 + 1)\mathbf{j}$ is conservative.

(b) Find a potential function $\phi(x, y)$ for $\mathbf{F}(x, y)$.

(c) Find the work done by the force field $\mathbf{F}(x, y)$ along the curve parameterized by $x = 2/t$, $y = t^2$, $1 \leq t \leq 2$.

Problem 9 (12 points)

(a) Find the double integral $\iint_R x^2\, dA$, where R is the region in the xy-plane between the x-axis and the graph of $y = 1 - x^2$.

(b) What is the physical meaning of this integral?

Problem 10 (8 points)

An inhomogeneous lamina of mass density $\rho(x, y) = x^2 + y^2$ fills the washer shaped region between the two disks of radius 1 and 2, both centered at the origin. Find the mass of the lamina.