EGR 265, Math Tools for Engineering Problem Solving
February 10, 2014, 50 minutes

TEST I

Name: .....

Problem 1	
Problem 2*	
Problem 3	
Problem 4	
Problem 5	
Problem 6	
Problem 7	
Total	

Problem 1 (4+4+4+4 Points)

Determine the order of the following ODEs. Also, state if they are linear or non-linear.

(a) $y''' + y' - xy = x$

(b) $y'' - y' = xy^2$

(c) $\dfrac{x}{y} = y'$

(d) $(xy')' = \cos x$

Problem 2* (8 Points Bonus)

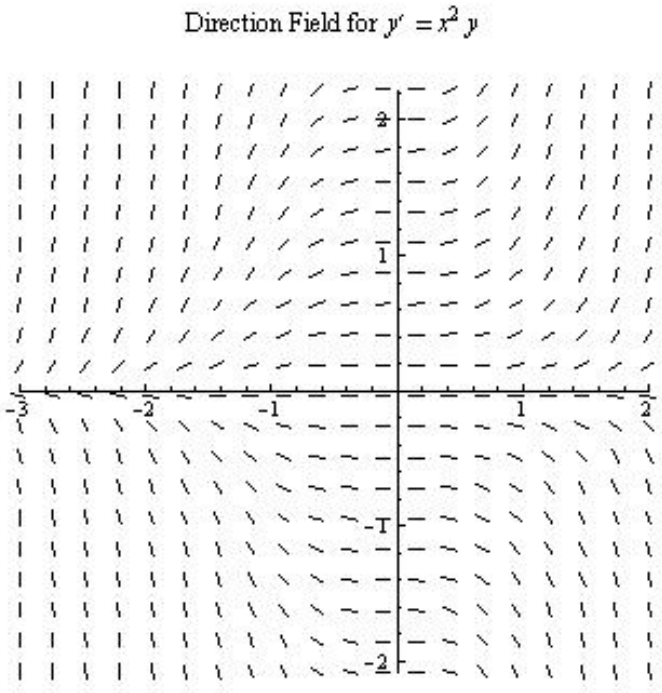
Find the values of the constant c for which the function $y = \frac{c}{x^2}$ is a solution of the differential equation

$$xy' + 3y = x^2y^2.$$

Warning: This DE is neither separable nor linear, thus we don't have a method to systematically find its solutions.

Problem 3 (4+4+2 Points)

Below the direction field for $y' = x^2 y$ is given.



(a) Sketch the solution $y(x)$ of $y' = x^2 y$ with initial value $y(0) = 1$. (Note: Do not solve the DE!)

(b) From your sketch determine $\lim_{x\to\infty} y(x)$ and $\lim_{x\to-\infty} y(x)$.

(c) Is it possible that two solution curves for $y' = x^2 y$ cross one another in the direction field? Justify!

Problem 4 (17 Points)

Solve the IVP

$$y' + xy^2 e^{x^2} = 0, \quad y(0) = 1.$$

Problem 5 (17 Points)

Solve the IVP

$$xy' + 2y = x, \quad y(1) = 1.$$

Problem 6 (15+5 Points)

(a) Find an implicit solution of the IVP

$$y' = \frac{x+1}{y}, \quad y(0) = -2.$$

(b) What is the correct explicit solution of the above IVP? Justify your answer!

Problem 7 (15+5 Points)

On Thanksgiving Day you walk into the kitchen and see that your little brother has already taken the turkey out of the oven. On the meat thermometer you see that its current core temperature is 120°F. After 10 minutes the temperature has dropped to 100°F. The air condition in your kitchen is set to 70°F.

(a) Use Newton's Law of Cooling to find the function $T(t)$ giving the core temperature of the turkey at time t. Set the time when you walked into the kitchen as $t = 0$. Here you don't have to evaluate logarithms, but you **have to solve the DE**.

Continued on next page!

(b) Your brother tells you that he took the turkey out 10 minutes before you walked into the kitchen. Determine if the turkey had reached a safe temperature of at least 165°F before it was taken out of the oven. For full credit, evaluate all the appearing numbers explicitly (which is possible without calculator) to give a definite answer.