MA 125, CALCULUS I
April 14, 2014

Name (Print last name first): .....

Student Signature: .....

TEST IV

No calculators are permitted!

PART I - Basic Skills

Part I consists of 7 questions. Clearly write your answer in the space provided after each question. You must explain your answers!!!

Each question is worth 7 points.

Question 1

Find the absolute minimum value of the function $f(x) = 2x^3 - 3x^2$ on the closed interval $[-2, 2]$. (Be sure to give the y-coordinate!)

Answer: .....

Question 2

Find the number c whose existence is guaranteed by the Mean Value Theorem for the function $f(x) = x^3 + x$ on the interval $[-1, 1]$.

Question 3

Find the critical number(s) of the function $f(x) = \dfrac{x^3 + 1}{x^3 - 1}$.

Answer: .....

Question 4

Find the open interval on which the function $g(x) = xe^x$ is decreasing. (Clearly indicate the end-points of your interval!)

Answer: .....

Question 5

Find the part of the x-axis on which the function $h(x) = \frac{1}{20}x^5 - \frac{1}{6}x^3$ is concave up.

Answer: .....

Question 6

Find the most general antiderivative of the function $f(x) = e^x + \frac{1}{1+x^2}$.

Answer: .....

Question 7

Find two positive numbers whose product is 9 and whose sum is minimal.

PART II - Problem Solving Skills

Points for each problem are indicated

Part II consists of 4 problems. You must show your work to get full credit. Displaying only the final answer (even if correct) without the relevant steps will not get full credit.

Problem 1 [16 points]

Suppose that the **derivative** of a function f is given by

$$f'(x) = (x-2)^4(x+1)^3$$

Answer all the following questions.

(a) Find all the critical numbers of the function f.

(b) On what interval(s) is the function f increasing? (Justify your answer!)

(c) On what interval(s) is the function f decreasing?

(d) Find the x-coordinates of all local max and min of the function. (Justify your answer!)

Problem 2 [10 points]

Find the dimensions of the open top box with a square base whose volume is $1000\,m^3$ so that its surface area is minimal.

Problem 3 [10 points]

An object moves along a straight line with acceleration

$$a(t) = 3t + 1.$$

Use antiderivatives to answer the following questions.

(a) Find the velocity function $v(t)$ of the object if its initial velocity $v(0) = 3$.

(b) Find the position function $s(t)$ of the object if its initial position $s(0) = 0$.

Problem 4 [16 points]

Consider the function f given by

$$f(x) = \frac{x^2}{x^2 - 4}.$$

Answer all the following questions.

(a) Find the x and y-intercept(s) of the curve.

(b) Find, if any, the vertical and horizontal asymptote(s) of the curve.

(c) Find the (open) interval(s) of increase, and the (open) interval(s) of decrease.

(d) Find, if any, all local maximum and minimum value(s). [Be sure to give the y-coordinate(s)!]

(g) Use the information from parts (a)–(d) above to sketch the graph.