EGR 265, Math Tools for Engineering Problem Solving
May 8, 2013, 10:45am to 1:15pm

Name (Print last name first): . . . .

Student ID Number: . . . .

Final Exam

Problem 1	
Problem 2	
Problem 3	
Problem 4 (incl. Bonus)	
Problem 5	
Problem 6 (incl. Bonus)	
Problem 7	
Problem 8	
Problem 9	
Problem 10	
Total (out of 100 + 8 Bonus)	

Problem 1 (8 points)

Find an explicit solution of the initial value problem

$$y' + x = -2xy, \quad y(0) = -1.$$

Problem 2 (10 points)

A liquid is heated to 120°F. It cools down according to Newton's law of cooling in a surrounding medium of temperature 60°F. The rate of cooling is $k = -0.5$.

(a) State the differential equation which governs the temperature of the medium at time t according to Newton's law of cooling.

(b) Solve this differential equation with the correct initial value (this can be done either as a separable or a linear equation).

(c) At what time has the temperature dropped to 80°F? (Logarithms do not need to be evaluated.)

Problem 3 (14 points)

Consider the second order differential equation

$$y'' - 3y' + 2y = 2\sin x. \tag{1}$$

(a) Find the general solution of the homogeneous equation corresponding to (1).

(b) Find a particular solution of the inhomogeneous equation (1).

(c) Solve the initial value problem given by (1) and initial conditions $y(0) = 3/5$, $y'(0) = 0$.

Problem 4 (10 points + 4 points bonus)

A mass of 100 kg stretches an undamped spring by 10 cm. Assume that $g = 10$ m/s^2.

(a) Find the spring constant k, including its correct unit.

(b) Set up the second order differential equation which governs the motion of the spring-mass system, choosing the x-axis to be oriented downwards. Find the general solution of this equation.

(c) Find the particular solution of the equation if the mass is released from the equilibrium position at a downward velocity of 50 cm/s.

(d) (Bonus) Suppose that in the above problem a damping force proportional to $\beta =$ 1000 kg/sec times the instantaneous velocity is added to the spring-mass system. Does the system become underdamped, critically damped or overdamped?

Problem 5 (10 points)

(a) Find the gradient of $f(x, y) = x(x + y)^3$.

(b) Evaluate the directional derivative of $f(x, y)$ at the point with coordinates $(1, 0)$ in the direction of the vector $\mathbf{v} = \mathbf{i} + 2\mathbf{j}$.

(c) Find a unit vector in the direction of steepest decrease of $f(x, y)$ at the point $(1, 0)$.

Problem 6 (8 points + 4 points bonus)

(a) Determine the equation of the tangent plane to the graph of $z = 2x^2 + xy + 3y$ through the point $(2, -1, 3)$.

(b) (Bonus) Is there a point (x, y) at which the tangent plane of $z = 2x^2 + xy + 3y$ is horizontal?

<u>Problem 7 (8 points)</u>

Find the line integral

$$\int_C xy^2\,ds,$$

where C is the straight line segment connecting the points $(-1,-1)$ and $(1,1)$.

Problem 8 (12 points)

(a) Verify that the force field $\mathbf{F}(x, y) = (x^3 + y)\mathbf{i} + (x + y^3)\mathbf{j}$ is conservative.

(b) Find a potential function $\phi(x, y)$ for $\mathbf{F}(x, y)$.

(c) Find the work done by the force field $\mathbf{F}(x, y)$ along the graph of the function $y = e^{x^2-1}$, $-1 \leq x \leq 1$.

Problem 9 (12 points)

A lamina of constant density $\rho(x, y) = 1$ is bounded by the curves $x = 0$, $y = 0$ and $y = 3 - x$. Use geometric considerations to simplify your work in the following.

(a) Sketch the lamina and find its mass.

(b) Find the lamina's centroid.

Problem 10 (8 points)

Use polar coordinates to find the double integral of the function $f(x, y) = (x^2 + y^2)^{10}$ over the disk of radius 1 centered at the origin.