EGR 265, Math Tools for Engineering Problem Solving
April 17, 2013, 50 minutes

TEST III

Name: ....

Problem 1	
Problem 2	
Problem 3	
Problem 4	
Problem 5	
Problem 6	
Problem 7	
Total	

Problem 1 (9+9 points)

(a) Let $f(x, y) = \ln(x^2 + y^2)$. Find its partial derivatives f_x and f_{xy}.

(b) Find the gradient of $g(x, y, z) = x \sin(yz)$.

Problem 2 (9+9 points)

(a) For the function $h(x, y) = (xy + 1)^2$ find a unit vector in the direction of steepest descent at the point $P(2, 1)$. Also, find the rate of descent in this direction.

(b) Find the directional derivative of $h(x, y)$ at $P(2, 1)$ in the direction of the point $Q(5, 3)$.

Problem 3 (12+6 points)

(a) Find an equation for the tangent plane to the level surface $x^2 - y^2 - 3z^2 = 5$ at the point with coordinates $(6, 2, 3)$.

(b) Also, find parametric equations for the normal line to $x^2 - y^2 - 3z^2 = 5$ at $(6, 2, 3)$.

Problem 4 (12 points)

Evaluate $\int_C ds$, where C is the curve parameterized by $x = t^2/2$, $y = t^3/3$, $1 \le t \le 2$.

Problem 5 (12 points)

Find the work done by the force field

$$F(x, y) = e^x \mathbf{i} + xy\mathbf{j}$$

along the straight line from the point $(0, 0)$ to the point $(1, 1)$.

Problem 6 (5+5 points)

Determine for each of the following force fields if it is conservative.

(a) $F(x, y) = \sin x \cos y \, \mathbf{i} - \cos x \cos y \, \mathbf{j}$

(b) $F(x, y) = (3x + y^2)\mathbf{i} + 2xy\mathbf{j}$

Problem 7 (12 points)

For the conservative force field $F(x, y)$ from Problem 6 find a potential function $\phi(x, y)$ and calculate the work done by the force field along the curve traced by the vector function $\mathbf{r}(t) = \cos t \, \mathbf{i} + \sin t \, \mathbf{j}$, $0 \le t \le \pi$.