EGR 265, Math Tools for Engineering Problem Solving
March 25, 2013, 50 minutes

Name: .....

TEST II

Problem 1	
Problem 2	
Problem 3	
Problem 4	
Problem 5	
Problem 6	
Problem 7*	
Total	

Problem 1 (20 points)

Solve the initial value problem

$$y'' - 4y' + 5y = 0, \quad y(0) = -2, \quad y'(0) = 2.$$

Problem 2 (20 points)

Find the general solution of

$$y'' - 2y' - 3y = 2x - 1.$$

Problem 3 (20 points)

Find the general solution of

$$y'' - 6y' + 9y = 2e^{-2x}.$$

Problem 4 (20 points)

A mass of 5 kilograms stretches an undamped spring by 98 centimeters.
(a) Find the value of the spring constant k using its correct metric unit.
(b) Find the angular frequency ω of free oscillations of the spring/mass-system.
(c) Find the equation of motion if the mass is released from rest at a position 10 centimeters above the equilibrium. Assume here that the positive x-direction is oriented downwards.
(d) Find the first positive time at which the mass passes through the equilibrium position.

Problem 5 (10 points)

Suppose that a damping force is added to the spring-mass system in Problem 4 which is proportional to the instantaneous velocity with damping coefficient $\beta = 20$ kg/sec. Does the resulting system become underdamped, critically damped, or overdamped? Justify your answer.

Problem 6 (10 points)

Find the largest interval centered around $x = 0$ in which the initial value problem

$$y'' + \frac{1}{x-2}y' + \frac{1}{x+2}y = 0, \quad y(0) = 1, y'(0) = 2$$

has a unique solution. Do not try to solve the DE! Use theoretical reasoning instead.

Problem 7* (5 points bonus)

Assume that the undamped spring mass system from Problem 4 is subjected to an exterior forcing function $F(t) = 5\sin(3t)$ Newtons. Without solving the DE, describe the phenomenon which you expect to see in its solutions. Give a reason.