SPRING 2013 — MA 227 — FINAL EXAM
SATURDAY, MAY 4, 2013

NAME: ____

THERE ARE 14 QUESTIONS, EACH WORTH 8 POINTS; 100 (OR MORE) POINTS IS EQUIVALENT TO 100% FOR THE EXAM. PARTIAL CREDIT IS AWARDED WHERE APPROPRIATE. SHOW ALL WORKING; YOUR SOLUTION MUST INCLUDE ENOUGH DETAIL TO JUSTIFY ANY CONCLUSIONS YOU REACH IN ANSWERING THE QUESTION.

1. Let $\mathbf{r}(t) = (t,\, t^2,\, t^3)$. Find normal plane at point $t = 2$.

2. Find the equation of the plane containing the points $(1, 1, 1)$, $(1, 1, -1)$ and $(-1, 2, 2)$.

3. Find the area of the parallelogram generated by the vectors $(2, 1, -1)$ and $(-1, 1, 2)$.

4. Let $f(x, y) = x\cos(y) - \frac{y}{x}$. Find all second partial derivatives: f''_{xx}, f''_{xy}, f''_{yy}.

5. Find local maximum, minimum and saddle points (if any) of the function

$$f(x, y) = x^2 - 2xy - y^2 + 4x - 1.$$

6. Let $z = e^x y + \frac{1}{y}$. Find equation of the tangent plane at point $(0, 1)$.

7. Find the maximum rate of change of $f(x, y) = x^3 - \sqrt{xy}$ at the point $(1, 1)$. In which direction does it occur?

8. Find the area of the region D bounded by $x = y^4$ and $y = x/8$.

9. Sketch the region of integration and change the order of integration:

$$\int_0^1 \int_x^{x^2+1} f(x, y)dydx.$$

10. Find the volume under the surface $z = x + y + 2$ and above the disc $x^2 + y^2 \leq 1$ in the xy plane. Use polar coordinates.

11. Acceleration of the particle is given by $\mathbf{a} = (-1, 0, 1)$. Find velocity and position of the particle as functions of time if at time $t = 0$ we have $\mathbf{v}(0) = (1, 0, 0)$ and $\mathbf{r}(0) = (1, 1, 1)$.

12. Find the absolute maximum and absolute minimum of the function $f(x, y) = x^2 - y^2 - 2x + 1$ on the region $0 \leq x \leq 2, \quad 0 \leq y \leq 1$. Be sure to provide coordinates of the points and the values of absolute maximum and minimum.

13. Using spherical coordinates, calculate the integral $\int\int\int_V z^2\, dxdydz$, where the region V is the half-ball: $\{x^2 + y^2 + z^2 \leq 4,\ x \geq 0\}$.

14. Calculate the integral

$$\int\int_D (x+y)\,dA,$$

where the region D is bounded by the lines $x+y=1,\ x+y=2,\ x-y=0,\ x-y=2.$ Use the change of variables $u=x+y,\ v=x-y.$