MA 227, CALCULUS III

Spring, 2013

Name (Print last name first): .....

Student Signature: .....

TEST 3

10 questions, 10 points each. SHOW ALL YOUR WORK!

Question 1

Find $\int\int_D x\,dxdy$, where D is bounded by $y = x^4$ and $y = x^2$.

Question 2

Find the volume under the surface $z = x$ and above the triangle in the xy plane with vertices $(0, 0)$, $(3, 0)$, $(1, 1)$.

Question 3

Sketch the region of integration and change the order of integration:

$$\int_0^1 \int_0^{y^2} f(x,y)dxdy.$$

Question 4

Use polar coordinates to find the volume under the plane $z = x - y + 2$ and above the half-disk $x^2 + y^2 \leq 1,\ y \leq 0$ in xy plane.

Question 5

Find the mass of the lamina that occupies the region:

$$D = \{(x, y)|\ x^2 + y^2 \leq 4,\ x \geq 0\}$$

and has the density function given by $\rho(x, y) = x^2 + y^2$.

Question 6

Evaluate the iterated integral $\int_0^1 dx \int_{x^2}^x dy \int_0^y dz$.

Question 7

Express the integral $\int\int\int_E f(x,y,z)dV$ as an iterated integral, where E is the solid above the region $D = \{(x,y) : x^3 \leq y \leq x^2\}$ in xy plane and below the plane $z = 2x + y$.

Question 8

Find $\int\int_D (x+2y)\,dxdy$, where D is bounded by $x-y=0$, $x-y=4$, $x+2y=1$, $x+2y=4$. Use change of variables $u=x-y$, $v=x+2y$.

Question 9

For the integral $\int\int_D f(x,y)\,dxdy$ consider the change of variables $u = xy^2$, $v = \frac{x}{y}$. Find inverse change of variables $x = x(u,v)$, $y = y(u,v)$ and calculate corresponding Jacobian. You DO NOT need to substitute it into integral.

Question 10

(a) Change $(R, \theta, \varphi) = (2,\ 2\pi/3,\ \pi/6)$ from spherical to rectangular coordinates.

(b) Change $(-\sqrt{3},\ 1,\ 2\sqrt{3})$ from rectangular to spherical coordinates.