MA 125-6C, CALCULUS I
April 29, 2013

Name (Print last name first): .....

Student Signature: .....

TEST IV

No calculators are permitted!

PART I - Basic Skills

Part I consists of 7 questions. Clearly write your answer in the space provided after each question.

Each question is worth of 6 points.

Question 1

Find the absolute minimum value of the function $f(x) = \frac{x^5}{5} - \frac{x^3}{3}$ on the closed interval $[0, 2]$. (Be sure to give the y-coordinate!)

Answer: .....

Question 2

Use the Mean Value Theorem on an a suitable interval to show that the equation $x^7 + x/2 = 0$ has at most one solution.

Question 3

Find the critical number(s) of the function $f(x) = \dfrac{2x^2 - 7}{x^2 - 4}$.

Answer: .....

Question 4

Find the open interval on which the function $g(x) = (x^2+x)e^x$ is decreasing. (Clearly indicate the end-points of your interval!)

Answer: .....

Question 5

Find the part of the x-axis on which the function $h(x) = \frac{1}{20}x^5 - \frac{2}{3}x^3$ is concave up.

Answer: .....

Question 6

Find the most general antiderivative of the function $f(x) = e^{2x} + \sqrt[3]{x} + \frac{1}{1+x^2}$.

Answer: .....

Question 7

Find two positive numbers whose product is 81 and whose sum is minimal.

PART II - Problem Solving Skills

Points for each problem are indicated

Part II consists of 4 problems. You must show your work to get full credit. Displaying only the final answer (even if correct) without the relevant steps will not get full credit.

Problem 1 [12 points]

Suppose that the derivative of a function f is given by

$$f'(x) = -(x-3)^6(x+1)^4$$

Answer all the following questions.

(a) Find all the critical numbers of the function f.

(b) On what interval(s) is the function f increasing? (Justify your answer!)

(c) On what interval(s) is the function f decreasing? (Justify your answer!)

Problem 2 [12 points]

Find the dimensions of the cylinder with bottom and lid whose volume is V_0 so that its surface area is minimal. [Hint: Volume $V = \pi r^2 h$ and surface area is $S = 2\pi r^2 + 2\pi r h$.]

Problem 3 [12 points]

An object moves along a straight line with acceleration

$$a(t) = -4\cos t + 3\sin t.$$

Use antiderivatives to answer the following questions.

(a) Find the velocity function $v(t)$ of the object if its initial velocity is $v(0) = 3$.

(b) Find the position function $s(t)$ of the object if its initial position is $s(0) = 0$.

Problem 4 [22 points]

Consider the function f given by

$$f(x) = \frac{2x^2}{x^2 - 1}.$$

Answer all the following questions.

(a) Find the x and y-intercept(s) of the curve.

(b) Find, if any, the vertical and horizontal asymptote(s) of the curve.

(c) Find the (open) interval(s) of increase, and the (open) interval(s) of decrease.

(d) Find, if any, all local maximum and minimum value(s). [Be sure to give the y-coordinate(s)!]

(e) Find the open interval(s) where the function is concave down, and the open interval(s) where it is concave up. [Hint: Factoring out might prove useful in your calculations!]

(f) Find the points of inflection point (if any).

(g) Use the information from parts (a)–(f) above to sketch the graph.