MA 125-6C, CALCULUS I

January 30, 2013

Name (Print last name first): .....

Student ID Number: ....

TEST I

PART I

Part I consists of 8 questions. Clearly write your answer (only) in the space provided after each question. You do not need not to show your work for this part of the test. No partial credit is awarded for this part of the test!

Each question is worth 5 points.

Question 1

Given that $\lim_{x\to a} f(x) = 7$ and $\lim_{x\to a} h(x) = -3$, find

$$\lim_{x\to a} \frac{4h(x)}{2h(x) + f(x)}.$$

Question 2

If $5x \leq f(x) \leq 3x^2 + x + 1$ for all $x > 1$, evaluate

$$\lim_{x\to 1^+} f(x)$$

.

Question 3

Evaluate the limit

$$\lim_{x\to\pi} \cos\left(\pi \sin\left(\frac{x}{2}\right)\right).$$

Question 4

Find the limit

$$\lim_{x\to\infty} \frac{-2x^6+3x^3+1}{3x^6-x^2+7x}.$$

Question 5

$$\lim_{x\to 0} \frac{4|x|}{x} = ?$$

Question 6

Calculate $\lim_{x\to 0^-} \frac{-2}{x^5}$.

Question 7

Calculate $\lim_{x\to 0} \dfrac{\cos^2(x) - 1}{5x^2}$.

Question 8

Calculte $\lim_{h\to 0} \dfrac{(4-h)^2 - 16}{h}$.

PART II

Part II consists of 5 problems. You must show your work on this part of the test to get full credit. Displaying only the final answer (even if correct) without the relevant steps will not get full credit.

Problem 1 (18 points)

Consider the function

$$f(x) = \begin{cases} 1 - 3x & \text{for } x \leq 2, \\ -x - 1 & \text{for } x > 2. \end{cases}$$

(a) Evaluate

$$\lim_{x \to 2^-} f(x).$$

(b) Evaluate

$$\lim_{x \to 2^+} f(x).$$

(c) Is this function continuous at $x = 2$? (Justify your answer!)

Problem 2 (18 points)

Sketch the graph of an example of a function f such that

$$\lim_{x\to 0^-} f(x) = -1, \qquad \lim_{x\to 0^+} f(x) = 3, \qquad f(0) = 2,$$

$$\lim_{x\to 3^-} f(x) = 0, \qquad \lim_{x\to 3^+} f(x) = -1, \qquad f(3) = 0.$$

Problem 3 (24 points)

(a) Evaluate the limit

$$\lim_{x\to 2} \frac{x^2-3x+2}{x^2+x-6}.$$

(b) Evaluate the limit $\lim_{x\to\infty}(\sqrt{2x+3}-\sqrt{2x})$.

(c) Show that the equation $\tan(x)+x+1=0$ has at least one solution in $(-\pi/2,\ \pi/2)$.