EGR 265, Math Tools for Engineering Problem Solving
May 7, 2012, 10:45am to 1:15pm

Name (Print last name first): ....

Student ID Number: ....

Final Exam

Problem 1	
Problem 2	
Problem 3	
Problem 4	
Problem 5	
Problem 6	
Problem 7	
Problem 8	
Problem 9	
Problem 10	
Total	

Problem 1 (8 points)

Find an explicit solution of the initial value problem

$$y' - 2x = 2xy, \quad y(1) = 0.$$

Problem 2 (10 points)

A liquid is heated to 100°F. It cools down according to Newton's law of cooling in a surrounding medium of temperature 50°F. The rate of cooling is $k = -0.5$.
(a) State the differential equation which governs the temperature of the medium at time t according to Newton's law of cooling.
(b) Solve this differential equation with the correct initial value (this can be done either as a separable or a linear equation).
(c) At what time has the temperature dropped to 75°F? (Logarithms do not need to be evaluated.)

Problem 3 (12 points)

Consider the second order differential equation

$$y'' - 4y' + 4y = 2e^{3x}. \tag{1}$$

(a) Find the general solution of the homogeneous equation corresponding to (1).
(b) Find a particular solution of the inhomogeneous equation (1).
(c) Solve the initial value problem given by (1) and initial conditions $y(0) = 0$, $y'(0) = -1$.

Problem 4 (12 points)

A mass of 100 kg stretches an undamped spring by 10 cm. Assume that $g = 10$ m/s^2. Include the correct units in all your answers below.
(a) Find the spring constant k and its correct unit.
(b) Set up the second order differential equation which governs the motion of the spring-mass system, choosing the x-axis to be oriented downwards. Find the general solution of this equation.
(c) Find the particular solution of the equation if the mass is released 50 cm below the equilibrium position from rest.
(d) What is the first positive time at which the mass returns to the equilibrium position?

Problem 5 (10 points)

(a) Find the gradient of $f(x, y) = ye^{xy}$.
(b) Evaluate the directional derivative of $f(x, y)$ at the point with coordinates $(0, 1)$ in the direction of the vector from $(0, 1)$ to $(1, 3)$.
(c) Find a unit vector in the direction of steepest increase of $f(x, y)$ at the point $(0, 1)$.

Problem 6 (10 points)

(a) Determine the equation of the tangent plane to the graph of $z = \frac{x}{x+y}$ through the point $(2, -1, 2)$.
(b) Also find parametric equations for the normal line to the graph from (a) at $(2, -1, 2)$.

Problem 7 (8 points)

Find the line integral

$$\int_C xy^2\, ds,$$

where C is a quarter of a unit circle centered at the origin and contained in the first quadrant, starting at (1, 0) and ending at (0, 1).

Problem 8 (11 points)

(a) Verify that the force field $F(x, y) = (2xy - y^2 + 1)\mathbf{i} + (x^2 - 2xy)\mathbf{j}$ is conservative.
(b) Find a potential function $\phi(x, y)$ for $F(x, y)$.
(c) Find the work done by the force field $F(x, y)$ along the curve $x = 2/t$, $y = t^2$, $1 \leq t \leq 2$.

Problem 9 (11 points)

A lamina of constant density $\rho(x, y) = 1$ is bounded by the curves $y = x^2$ and $y = 1$.
(a) Find the lamina's mass.
(b) Find the lamina's centroid. Use geometric considerations to simplify your work.

Problem 10 (8 points)

Rewrite the function $f(x,y) = x + y$ using polar coordinates and find its integral over the quarter disk of radius 1 in the first quadrant.