EGR 265, Math Tools for Engineering Problem Solving
February 6, 2012, 50 minutes

Name (Print last name first): ....

Student ID Number: ....

TEST I

Problem 1

Determine the order of the following ODEs. Also, state if they are linear or non-linear. (4P+4P+4P+4P)

(a) $yy'' = \cos x$

(b) $y^4 - y''' = e^x y$

(c) $y^{(4)} - y''' = e^x y$

(d) $\dfrac{y - \cos x}{y'} = e^x$

Problem 2 (8P+4P)

(a) Find all values of C such that $y = Cx^2$ is a solution of

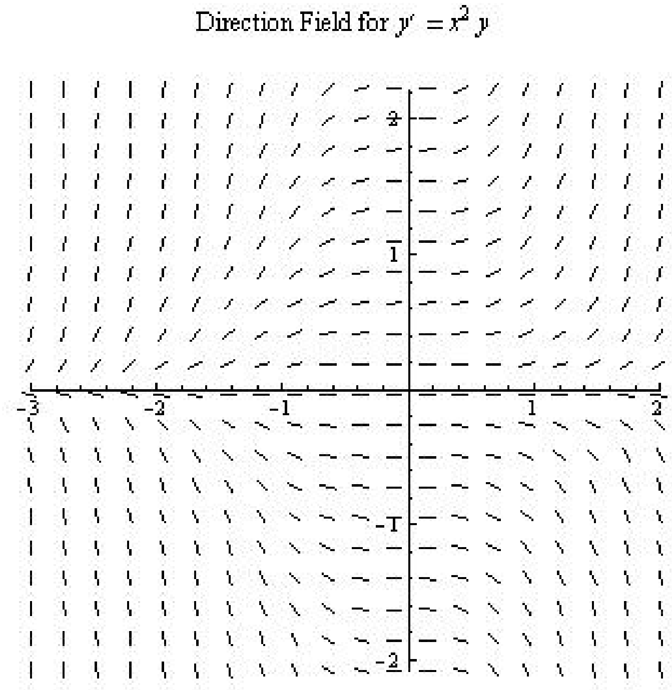
$$x^4 y' - xy^2 + 8x^5 = 0.$$

(b) Find the unique solution of the initial value problem

$$x^4 y' - xy^2 + 8x^5 = 0, \quad y(2) = -8.$$

Problem 3

Below the direction field for $y' = x^2 y$ is given.



(a) Sketch the solution $y(x)$ of $y' = x^2 y$ with initial value $y(0) = 1$. (Note: Do not solve the DE!) (4P)

(b) From your sketch determine $\lim_{x\to\infty} y(x)$ and $\lim_{x\to-\infty} y(x)$. (4P)

(c) Is it possible that two solution curves for $y' = x^2 y$ cross one another in the direction field? Justify! (2P)

Problem 4

Solve the IVP (15P)

$$4y^3y' = \cos x, \quad y(\pi/2) = 2.$$

Problem 5

Solve the IVP (15P)

$$\frac{y'}{x} + 2y = 1, \quad y(0) = 1$$

Problem 6

Solve the IVP (15P)

$$2x - y = y', \quad y(0) = 3$$

Problem 7

Water is heated to 150°F and then placed outside on a day when the temperature is 20°F. Within one minute the temperature drops by 40°F.
Note: Your answers to the questions below will contain natural logarithms which do not need to be evaluated.

(a) Based on Newton's law of cooling, write down the IVP for this problem using an unknown cooling rate k. (4P)

(b) Solve the IVP and determine k by using information provided in the problem. (8P)

(c) At what time does the water freeze? (5P)