Calculus II, Exam III, Spring 2012

Name:______________________

Student signature:______________________

Show all your work and give reasons for your answers. Good luck!

Part I

Each problem in part I is worth 7 points; You **must** justify your answers!!

Evaluate the following integrals

(1) Find the area bounded by the graphs of the functions $y = \cos(x) + 5$ and $y = x$ between $x = \pi/2$ and $x = \pi$.

(2) **Set up** an integral for the solid of revolution obtained by rotating the area described in problem 1 around the **x**-axis.

(3) **Set up** an integral for the solid of revolution obtained by rotating the area described in problem 1 around the **y**-axis.

(4) Determine if the improper integral $\int_1^\infty \frac{1}{x^2}\,dx$ is convergent or divergent. If it is convergent, evaluate the integral.

(5) Find the arc length of the curve $\vec{r}(t) =< \sin(t), \cos(t) >$ for $\pi/2 \leq t \leq \pi$.

(6) **Set up** an integral for the arc length of the graph of the function $y = f(x) = (x^2 + 1)^3$ for $0 \leq x \leq 1$.

(7) Find the work done in moving a mass of $7\,kg$ a distance of $2m$ horizontally and $3\,m$ vertically upward.

Part II

Each problem in part II is worth 13 points.
Justify all your work for full credit!!

In the next two problems **set up** integrals for the volume of the solid obtained by rotating the area bounded by $y = f(x) = x^3 + x + 3$, $y = g(x) = \sin(x)$, $x = 0$ and $x = \pi/2$ about the indicated axis.

1. Rotate about the line $x = -2$.

2. Rotate the above region about the line $y = -2$.

3. Find the volume of the solid whose cross sections perpendicular to the x-axis are round disks with their diameter stretching from the graph of $y = f(x) = 2x - 3$ to the graph of $y = g(x) = x^2 + 2x$ for $0 \leq x \leq 1$.

4. Find the work done in pumping all the water out of a half full conical tank (with vertex up) of height $h = 8\,m$, radius $r = 6\,m$ (i.e., the water in the tank is up to level $4\,m$ from the bottom). Use $g \approx 10\ m/sec^2$ and density of water $\rho = 1,000\ kg/m^3$.