EGR 265, Math Tools for Engineering Problem Solving
April 18, 2011, 50 minutes

Name (Print last name first): .....

TEST III

$P1$:	
$P2$:	
$P3$:	
$P4$:	
$P5$:	
$P6$:	
$P7$:	

Problem 1 (9+9 points)

(a) Let $f(x, y) = 2x^2y^2 + 3y^3$. Find $f_{xx} + f_{yy}$.

(b) For the function $g(x, y) = xe^{xy}$ find g_x, g_y and g_{yx}.

Problem 2 (9+9 points)

(a) For the function $h(x, y) = \ln(x^2 + y^2)$ find its direction **and** rate of steepest descent at the point $P(1, 1)$.

(b) Find the directional derivative of $h(x, y)$ at $P(1, 1)$ in the direction of the vector $\mathbf{v} = -4\mathbf{i} + 3\mathbf{j}$.

Problem 3 (12+6 points)

(a) Find an equation for the tangent plane to the graph of $z = \sqrt{x+y}$ at the point $(1, 0, 1)$.

(b) Also, find parametric equations for the normal line to the graph of $z = \sqrt{x+y}$ at $(1, 0, 1)$.

Problem 4 (12 points)

Evaluate $\int_C xy^4\,ds$, where the curve C is the quarter circle $x = 2\cos t$, $y = 2\sin t$, $0 \le t \le \pi/2$.

Problem 5 (12 points)

Find the work done by the force field

$$F(x, y) = ye^{x^3}\mathbf{i} + 4xy\mathbf{j}$$

along the curve C given by the graph of $y = x^2$, $0 \le x \le 1$.

Problem 6 (5+5 points)

Determine for each of the following force fields if it is conservative.

(a) $F(x, y) = ye^x\mathbf{i} + xe^y\mathbf{j}$

(b) $F(x, y) = 2xy\mathbf{i} + (x^2 + y)\mathbf{j}$

Problem 7 (12 points)

For the conservative force field $F(x, y)$ from Problem 6 find a potential function $\phi(x, y)$ and calculate the work done by the force field along the curve traced by the vector function $\mathbf{r}(t) = t^2 \sin(t)\mathbf{i} + t^2 \cos(t)\mathbf{j}$, $0 \leq t \leq \pi$.