EGR 265, Math Tools for Engineering Problem Solving
March 7, 2011, 50 minutes

Name (Print last name first): .....

Student ID Number: ....

TEST II

$P1$:	
$P2$:	
$P3$:	
$P4$:	
$P5$:	
$P6$:	

Problem 1 (20 points)

Solve the initial value problem

$$y'' - 6y' + 9y = 0, \quad y(0) = 1, \quad y'(0) = -1.$$

Problem 2 (20 points)

Find the general solution of

$$y'' - 2y' + 10y = 10x + 3.$$

Problem 3 (20 points)

Find the general solution of

$$y'' + 2y' - 3y = 8e^x.$$

Problem 4 (20 points)

A mass of 2 kilograms stretches an undamped spring by 98 centimeters.
(a) Find the value of the spring constant k. Include the correct unit of k (using the mks-system).
(b) Find the frequency $\frac{\omega}{2\pi}$ of free oscillations of the spring/mass-system.
(c) Find the equation of motion if the mass is released from rest at a position 30 centimeters below the equilibrium. Assume here that the positive x-direction is oriented downwards.
(d) Find the first positive time at which the mass passes through the equilibrium position. (Note: In the example considered here, there is a simpler way to answer this than the method used in class.)

Problem 5 (10 points)

Suppose that a damping force is added to the spring/mass system in Problem 4 which is numerically equal to 10 times the instantaneous velocity. Does the resulting system become underdamped, critically damped, or overdamped? Justify your answer.

Problem 6 (10 points)

Consider the motion of a falling body near the surface of the Earth. The velocity v of the body is described by the differential equation

$$v'(t) = g - \frac{k}{m}v(t), \tag{1}$$

where $g = 9.8$ m/s is the acceleration due to gravity, m is the mass of the body, and k is a constant of proportionality for air resistance.

(i) Find the solution of the differential equation (1) satisfying the initial value $v(0) = v_0$.

(ii) Find the terminal velocity $v_\infty = \lim_{t\to\infty} v(t)$.