MA-227, Calculus III
Common Final Test
April 29, 2011

Time available: 150 min
Each problem is 10 points

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1. Determine whether or not

$$\vec{F}(x,y) = (e^y - y\sin x)\vec{i} + (xe^y + \cos x)\vec{j}$$

is a conservative vector field, and if yes, find a potential function f for it.

2. Use Green's Theorem to evaluate $\int_C \vec{F} \cdot d\vec{r}$, where

$$\vec{F}(x,y) = \langle \sin x - 3x^2y, \, \cos y + 3xy^2 \rangle,$$

and C is the circle $x^2 + y^2 = 4$ oriented counterclockwise.

3. The lamina D is defined by the inequalities $0 \leq x \leq 1$, $0 \leq y \leq 1$, and its mass density function is given by $\rho(x, y) = x^3 + y^3$. Compute its mass and the center of mass.

4. The solid B lies inside the cylinder $x^2+y^2=1$ and inside the sphere $x^2+y^2+z^2=9$. Calculate its volume.

5. Evaluate the integral by reversing the order of integration.

$$\int_0^{16} \int_{y^{1/4}}^{2} e^{x^5}\, dx dy.$$

6. Calculate the integral

$$\int_1^2 \int_1^2 (\frac{x^6}{y} + \frac{y^6}{x}) dy dx.$$

7. Find the minimum and maximum values of the function $f(x, y, z) = 5y(x + z)$ subject to the constraints $xy = 1$ and $y^2 + z^2 = 3$.

8. We know that x, y, and z are positive numbers the sum of which is equal to 1. Maximize the value of xyz^4.

9. Find the points on the ellipsoid $4x^2 + y^2 + z^2 = 1$ where the normal line is parallel to the line connecting the points $(2, -1, -2)$ and $(3, 0, -1)$.

10. Let $z = y^2 \sin x$, $x = tuv$, $y = u^2 + tv$. Find $\partial z/\partial t$, $\partial z/\partial u$, and $\partial z/\partial v$ when $t = 3$, $u = 2$, $v = 0$.

11. Find the work done by the force field $\vec{F}$ moving an object from $P(1,1)$ to $Q(5,5)$, where

$$\vec{F}(x,y) = x^{5/2}\vec{\imath} + x\sqrt{y}\vec{\jmath}.$$

12. Use Green's Theorem to evaluate the line integral

$$\int_C xe^{-2x}dx + (x^5 + \frac{5}{3}x^3y^2)dy,$$

where C is the positively oriented boundary between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.