Calculus II, Final Exam, Spring 2011

Name:_____

Show all your work and give reasons for your answers. Good luck!

(1) (5 points) Find the angle between the vectors $< 3, -2, 0 >$ and $< 1, -1, 2 >$. You may express your answer using $\arccos(x)$.

(2) (5 points) Find the parametric equation of the line perpendicular to the plane $3x - y + 4z = 5$ through the point $(-1, 2, -3)$.

(3) (5 points) Find the area of the region bounded by the curves $x = 0$, $x = \pi$, $y = -x^3 - 1$ and $y = \sin(x)$.

(4) (5 points) Evaluate $\int x \sin(x^2)\, dx$.

(5) (5 points) Evaluate $\int \frac{x^3+\sqrt{x}}{x}\, dx$.

(6) (5 points) Evaluate $\int x \sin(x)\, dx$.

(7) (5 points) Express $f(x) = \frac{x}{1+x^2}$ as a power series. **Also** state the interval of convergence.

(8) (5 points) Use series to approximate $\cos(\frac{1}{10})$ with an error less than 10^{-6}. [You do not need to add the terms in the sum.]

(9) (5 points) Evaluate $\int \sin^3(x)\,dx$.

(10) (5 points) Find the interval and radius of convergence of the power series $\sum_{n=0}^{\infty} \left(\frac{x}{3}\right)^n$.

(11) Evaluate the following integrals:

(a) (6 points) $\int \frac{\sin(\sqrt{x})}{\sqrt{x}}\, dx$.

(b) (8 points) Obtain the partial fraction decomposition of $\frac{1}{x(x+1)^2}$. Also compute the constants.

(c) (5 points) Evaluate $\int \frac{1}{x(x+1)^2}\, dx$.

(12) (12 points) **Set up (but do not evaluate)** an integral for the volume of the solid of revolution obtained by rotating the region bounded by the curves $x = 0$, $x = 1$, $y = x^2 + 5$ and $y = x - 1$ around the line $x = -3$.

(13) (10 points) Find the work done in pumping all the water out of a full cone (with vertex down) of radius $5\,m.$ and height $3\,m.$ [You may use the approximation $g \approx 10\,m/sec^2$ and $\rho = 1000\,kg/m^3$.]

(14) (10 points) Find the distance from the point $(1, 2, 3)$ to the plane $2x - y + z = 4$.

Scratch paper