Calculus II, Exam III, Spring 2011

Name:____________________

Student signature:____________________

Show all your work and give reasons for your answers. Good luck!

Part I

Each problem in part I is worth 6 points; Show your work!!

Evaluate the following integrals

(1) Find the area bounded by the graphs of the functions $y = x^3 + 5$ and $y = \sin(x)$ between $x = 0$ and $x = \pi/2$.

(2) **Set up** an integral for the solid of revolution obtained by rotating the area described in problem 1 around the **x**-axis.

(3) **Set up** an integral for the solid of revolution obtained by rotating the area described in problem 1 around the **y**-axis.

(4) Determine if the improper integral $\int_1^\infty \frac{1}{x^3}\,dx$ is convergent or divergent. If it is convergent, evaluate the integral.

(5) Find the arc length of the curve $\vec{r}(t) =< \cos(t), \sin(t) >$ for $0 \leq t \leq \pi/2$.

(6) **Set up** an integral for the arc length of the graph of the function $y = f(x) = \sin(x)$ for $0 \leq x \leq \pi$.

(7) Find the work done in stretching a spring $1\,m$ from its rest position if it takes a force of $500\,N$ to stretch it $2m$ from its rest position.

(8) Find the work done in moving a mass of $5\,kg$ a distance of $1m$ horizontally and $1\,m$ vertically upward.

Part II

Each problem in part II is worth 13 points.
Justify all your work for full credit!!

In the next two problems **set up** integrals for the volume of the solid obtained by rotating the area bounded by $y = f(x) = x^2 + 2$, $y = g(x) = \sin(x)$, $x = 0$ and $x = \pi/2$ about the indicated axis.

1. Rotate about the line $x = -3$.

2. Rotate about the line $y = -3$.

3. Find the volume of the solid whose cross sections perpendicular to the x-axis are round disks with their diameter stretching from the graph of $y = f(x) = \sqrt{x}$ to the graph of $y = g(x) = x^2$ for $0 \leq x \leq 1$.

4. Find the work done in pumping all the water out of a half full conical tank (with vertex down) of height $h = 5\,m$, radius $r = 4\,m$ (i.e., the water in the tank is up to level $2\,m$ from the bottom). Use $g \approx 10\ m/sec^2$ and density of water $\rho = 1,000\ kg/m^3$.