Calculus II, Exam I, Spring 2011

Name:____________________

Student signature:____________________

Show all your work and give reasons for your answers. Good luck!

Part I

Each problem in part I is worth 5 points; Show your work!!

(1) Find the angle between the vectors $\vec{a} = < 0, 1, 2 >$ and $\vec{b} = < -1, 2, -3 >$. (You may express your answer in terms of arccos.)

(2) Find the equation of the plane perpendicular to the line $x = 2 + t$, $y = 1 - 2t$ and $z - 1 - 3t$ which passes through the point $(-1, -1, -3)$.

(3) Find the area of the parallelogram spanned by the vectors $< 1, 0, 1 >$ and $< -1, 2, 1 >$.

(4) If $\vec{a} = < -2, -1, 3 >$ and $\vec{b} = < 2, 1, 3 >$ find the component $\text{com}_{\vec{b}}(\vec{a})$.

(5) if $\vec{u} = < 1, 0, 1 >$ and $\vec{v} = < 2, 5, -2 >$ is $\vec{u}$ perpendicular to $\vec{v}$? (You **must** justify your answer.)

(6) If $\vec{r}(t) = < \sin(t),\ t^3,\ e^t >$, find $\lim_{t \to \pi} \vec{r}(t)$.

(7) If $\vec{r}(t) = < e^t,\ t\cos(t),\ t^2 >$ find the derivative $\vec{r}(t)'$.

(8) If $\vec{r}(t) = < e^t,\ t\cos(t),\ t^2 >$, find the unit tangent vector $T(t)$.

(9) Find the distance between the panes $2x + y - z = 3$ and $4x + 2y - 2z = 10$.

(10) Are the lines $\frac{x-1}{1} = \frac{y+2}{5} = \frac{z-2}{-2}$ and $\frac{x+4}{2} = \frac{y-7}{10} = \frac{z+11}{-4}$ parallel? (You **must** justify your answer.)

(11) Are the vectors $< 1, 0, 2 >$, $< 2, 3, 1 >$ and $< 1, 3, -1 >$ coplanar (You must justify your answer!)

Part II

(1) (a) [**5 points**] Find the area of the triangle with vertices $P = (1, 2, 1)$, $Q = (2, 2, 3)$ and $R = (3, 1, 1)$.

(b) [**5 points**] Determine if the vectors $< 1, 0, 1 >$, $< 2, -1, 1 >$ and $< -1, 2, 1 >$ are coplaner. (You must justify your answer.)

(2) [**15 points**] Given the lines:

$$\ell_1 = \begin{cases} x = -1 + 2t \\ y = 1 + t \\ z = 4 - t \end{cases} \quad \text{and} \quad \ell_2 = \begin{cases} x = -1 + t \\ y = 2 + 3t \\ z = -1 - 2t \end{cases}$$

determine if they are skew or not. If they are skew, find their distance. If not, find the point of intersection.

(3) **[10 points]** Find the line of intersection of the planes $x - 2y + 3z = 1$ and $2x + y - z = 4$.

(4) Let $\vec{r}(t) = < t\sin(t),\ (t^2+1)^5,\ \ln(t) >$ be the position of a fly at time t, find

(a) **[5 points]** The velocity vector $\vec{v}(t)$ at time $t = \pi$.

(b) **[5 points]** The speed at time $t = \pi$.

(c) [**5 points**] The unit tangent vector $\vec{T}(t)$ at time $t = \pi$.

(5) [**Bonus: 5 points**] Assume that $|\vec{r}(t)| = c$ is constant show that $\vec{r}(t)$ is perpendicular to $\vec{r}(t)'$. (Hint use the fact that $\vec{r}(t) \cdot \vec{r}(t) = c^2$ is constant and, hence $0 = \frac{d}{dt}\left[\vec{r}(t) \cdot \vec{r}(t)\right]$.) Do you see a geometric interpretation of this fact?