MA 125 - 6C, CALCULUS I
March 28, 2011

Name (Print last name first): .....

Student Signature: .....

TEST III

No calculators are allowed!

PART I

Part I consists of 6 questions. Clearly write your answer (only) in the space provided after each question. You do not need not to show your work for this part of the test. No partial credit is awarded for this part of the test!

Question 1

Differentiate the function $y = x\ln(x) - x$.

Answer: .....

Question 2

Find the derivative of the following quotient $f(x) = \dfrac{e^x}{x}$.

Answer: .....

Question 3

Differentiate the function $y = e^{2x}\sin(x)$.

Answer: .....

Question 4

Suppose $h(x) = \dfrac{\ln(x)}{g(x)}$ where $g(1) = -5$ and $g(x)$ is differentiable. Find the numerical value of $h'(1)$.

Answer: .....

Question 5

Find the derivative of the function $y = \arctan(x^2)$.

Answer: .....

Question 6

Evaluate $\lim\limits_{x\to 0} \dfrac{e^{3x}-1}{x}$.

Answer: .....

PART II

Each problem is worth 14 points.

Part II consists of 5 problems. You must show your work on this part of the test to get full credit. Displaying only the final answer (even if correct) without the relevant steps will not get full credit.

Problem 1

(1) Find the linear approximation of the function $g(x) = \sqrt{1-x}$ at $a = 0$, and then use it to approximate the number $\sqrt{0.98}$.

(Hint: To approximate $\sqrt{0.98}$ make sure that you use the correct value of x in your linearization!)

(2) Use Newton's method with initial approximation $x_1 = 1$ to find x_2, the second approximation to the root of the equation

$$x^4 - x - 2 = 0.$$

Problem 2

The radius of a circular plate is given as 10 inches with a maximum error in measurement of 0.01 inches.

(1) Use differentials to estimate the maximum error in the calculated area of the plate (in square inches).

(2) What is the relative error?

(3) What is the percentage error?

Problem 3

(1) Differentiate the function, and simplify completely by expressing your answer as a single fraction.

$$g(x) = \ln\left(\frac{e^x + 1}{e^x - 1}\right).$$

(Hint: Properties of the logarithmic function might prove useful here!)

(2) Differentiate the function. (Simplify your answer!)

$$y = \ln\left(e^x \cos(x)\right).$$

Problem 4

(a) Use logarithmic differentiation to find the derivative of the function

$$y = x^x.$$

(b) Differentiate the function $y = \arctan(\sin(x))$.

Problem 5

(1) Evaluate

$$\lim_{x\to\infty} x^3 e^{-x}.$$

(2) If $f'(x)$ is continuous, $f(5) = 0$ and $f'(5) = 9$, evaluate

$$\lim_{x\to 0} \frac{f(5+4x) + f(5+6x)}{x}.$$

(Your answer must be a real number!)

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