

MA 125 - 6C, CALCULUS I

February 02, 2011

Name (Print last name first):

Student Signature:

TEST I

No calculators are allowed!

PART I

Part I consists of 6 questions. Clearly write your answer (only) in the space provided after each question. You do not need not to show your work for this part of the test. No partial credit is awarded for this part of the test!

Each question is worth 5 points.

Question 1

The function

$$f(x) = \begin{cases} x - 1 & \text{for } x < 2, \\ x^2 - 5 & \text{for } x \geq 2. \end{cases}$$

has a discontinuity at the number $x = 2$. What kind of discontinuity is it?

Answer:

Question 2

If $f(x)$ and $g(x)$ are continuous functions with $f(-3) = 5$ and the following limit $\lim_{x \rightarrow -3} [4f(x) - g(x)] = 11$. Find $g(-3)$.

Answer:

Question 3

Evaluate the limit $\lim_{x \rightarrow 0} \frac{\sin(9x)}{3x}$.

Answer:

Question 4

If $6x \leq g(x) \leq 3x^4 - 3x^2 + 6$ for all x , evaluate

$$\lim_{x \rightarrow 1} g(x)$$

if the limit exists.

Answer:

Question 5

Evaluate the limit

$$\lim_{x \rightarrow \infty} \frac{x^3 + 3x}{2x^3 - x^2 + 8}$$

Answer:

Question 6

Given that $f(1) = 2$ and $f'(1) = -1$, find an equation of the tangent line to the graph of $y = f(x)$ at $x = 1$.

Answer:

PART II

Each problem is worth 14 points.

Part II consists of 5 problems. You must show your work on this part of the test to get full credit. Displaying only the final answer (even if correct) without the relevant steps will not get full credit.

Problem 1

Consider the function

$$f(x) = \begin{cases} \frac{x^2 - 1}{x - 1} & \text{for } x < 1, \\ 2 & \text{for } x = 1, \\ \frac{2}{x - 1} & \text{for } x > 1. \end{cases}$$

(a) Evaluate

$$\lim_{x \rightarrow 1^-} f(x).$$

(b) Evaluate

$$\lim_{x \rightarrow 1^+} f(x).$$

(c) Is this function continuous at $x = 3$? (Justify your answer!)

Problem 2

Consider the curve

$$y = \frac{6x^2 + x - 7}{x^2 - 4x + 3}.$$

(a) Find, if any, the horizontal asymptote(s) of this curve. (Justify your answer!)

(b) Find, if any, the vertical asymptote(s) of this curve. (Justify your answer!)

Problem 3

(a) Evaluate the limit

$$\lim_{x \rightarrow 3} \frac{x^2 - 2x - 3}{x^2 - 5x + 6}$$

(b) Evaluate the limit

$$\lim_{x \rightarrow 0} \frac{\sqrt{2+x} - \sqrt{2}}{x}$$

Problem 4

(a) For what (numerical) value of the constant c is the function

$$f(x) = \begin{cases} cx^2 + 2x & \text{if } x > 3 \\ x^3 - cx & \text{if } x \leq 3 \end{cases}$$

continuous on $(-\infty, \infty)$? (Justify your answer!)

(b) The following limit

$$\lim_{x \rightarrow 1} \frac{\frac{4}{x+1} - 2}{x-1}$$

represents the derivative of some function $f(x)$ at some number a . Explicitly find an appropriate function $f(x)$ and a number a .

Problem 5

Let $f(x) = \frac{4}{x+1}$.

(a) Use **the limit definition of the derivative** to find $f'(1)$.

(b) Find an equation of the tangent line to the curve $f(x) = \frac{4}{x+1}$ at the point $(1, 2)$.

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