EGR/MA 265, Math Tools for Engineering Problem Solving
March 10, 2010, 50 minutes

Name (Print last name first): . . . .

Student ID Number: . . . .

$$\boxed{\textbf{TEST II}}$$

Problem 1 (20 points)

Solve the initial value problem

$$y'' - 4y' + 4y = 0, \quad y(0) = -1, \quad y'(0) = 1.$$

Problem 2 (20 points)

Find the general solution of
$$y'' + 2y' + 10y = 14 + 20x.$$

<u>Problem 3 (20 points)</u>

Find the general solution of
$$y'' + 5y' + 6y = 20\cos(x).$$

Problem 4 (20 points)

A mass of 2 kilograms stretches an undamped spring by 196 centimeters.

(a) Find the value of the spring constant k. Include the correct unit of k (using the mks-system).

(b) Find the frequency $\frac{\omega}{2\pi}$ of free oscillations of the spring/mass-system.

(c) Find the equation of motion if the mass is released from rest at a position 100 centimeters below the equilibrium. Assume here that the positive x-direction is oriented downwards.

(d) Find the first positive time at which the mass passes through the equilibrium position. (Note: In the example considered here, there is a simpler way to answer this then the method used in class.)

Problem 5 (10 points)

Suppose that a damping force is added to the spring/mass system in Problem 4 which is proportional to the instantaneous velocity with damping coefficient $\beta = 0.5$ kg/sec. Does the resulting system become underdamped, critically damped, or overdamped? Justify your answer.

Problem 6 (10 points)

Find the largest interval containing $x = 0$ for which the IVP

$$y'' + \frac{1}{\sqrt{1-x}} \ y = \frac{1}{x^2 - 4}, \quad y(0) = 0, \quad y'(0) = 1$$

has a unique solution.

SCRATCH PAPER

(Scratch paper will not be graded!)

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