EGR/MA 265, Math Tools for Engineering Problem Solving
February 08, 2010, 50 minutes

Name (Print last name first): .....

Student ID Number: .....

TEST I

Problem 1 (16P)

Determine the order of the following ODEs. Also, state if they are linear or non-linear. (4P+4P+4P+4P)

(a) $y' + \sin(y) = x$

(b) $y^{(6)} - y^{(3)} = \cos(xy)$

(c) $yy'' = e^x$

(d) $\dfrac{y - \sin(x)}{x^2 y'} = e^x$

Problem 2 (12P)

(a) Which of the following functions are solutions of $x^4 y' + 2xy^2 = 4x^5$? (8P)

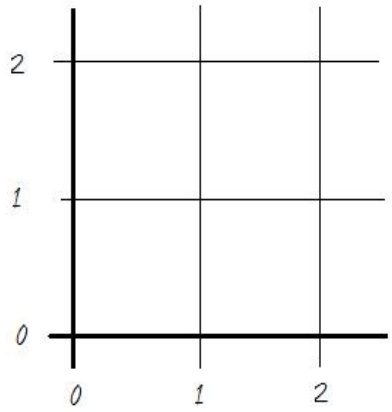
$$y_1 = -x^2, \quad y_2 = x, \quad y_3 = x^2, \quad y_4 = -2x^2.$$

(b) Which of the functions from part (a) solve the initial value problem $x^4y' + 2xy^2 = 4x^5$, $y(0) = 0$? (4P)

(c)* (Bonus) Does your answer to part (b) agree with the content of the Existence and Uniqueness Theorem for first order ODEs? If yes, why? If no, why not? (5P*)

Problem 3 (12P)

(a) In the 3×3-grid of points $x = 0, 1, 2$ and $y = 0, 1, 2$ provided in the figure below draw a direction field for $y' = x^2(y - 2)$. (8P)



(b) Without solving the DE, use the direction field to read off the solution of the IVP $y' = x^2(y - 2)$, $y(1) = 2$. (4P)

Problem 4 (12P)

Solve the IVP

$$y' = x^2(y-2), \quad y(1) = 1.$$

Problem 5 (14P)

Solve the IVP

$$\frac{y'+y}{x} = 1, \quad y(0) = 3$$

Problem 6 (12P)

Solve the IVP

$$y' - y^2 \sin(x) = 0, \quad y\left(\frac{\pi}{3}\right) = 2$$

Problem 7 (12P)

The mass of a radioactive material is given by $m(t)$, where the time t is measured in years and the mass in grams. An initial mass of $m(0) = 100$ grams decays at a constant rate $k = m'(t)/m(t)$. After 1 year 80 grams of the material are left.

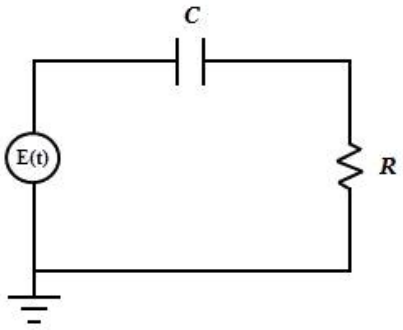
(Note: Your answers will contain natural logarithms which do not need to be evaluated.)

(a) Find the decay rate k by solving the differential equation for $m(t)$. (8P)

(b) Find an expression for the time t_h at which only 50 grams of the material are left (t_h is the so-called *half-life* of the material). (4P)

Problem 8 (10P)

In the electrical circuit below one has $R = 100$ ohms, $C = 0.01$ farads and a constant electromotive force of $E(t) = 100$ volts.



(a) Write down the DE for the charge $q(t)$ and solve it to find a closed form solution for $q(t)$ in Coulombs given that $q(0) = 0$. (6P)

(b) Find a formula for the current $i(t)$ in amperes. (4P)

SCRATCH PAPER

(Scratch paper will not be graded!)

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