SPRING 2010 — MA 227 — FINAL EXAM
FRIDAY APRIL 30, 2010

NAME: ____

THERE ARE 11 QUESTIONS, EACH WORTH 10 POINTS; 100 (OR MORE) POINTS IS EQUIVALENT TO 100% FOR THE EXAM. PARTIAL CREDIT IS AWARDED WHERE APPROPRIATE. SHOW ALL WORKING; YOUR SOLUTION MUST INCLUDE ENOUGH DETAIL TO JUSTIFY ANY CONCLUSIONS YOU REACH IN ANSWERING THE QUESTION.

1. (a) Find the equation of the plane containing the points $(1, 2, 2)$, $(1, 1, -1)$ and $(-1, 2, 1)$.
   (b) Let $\mathbf{r}(t) = (4t^{1/4},\ e^{t^2-1},\ 2t)$. Find the unit tangent vector at the point on the curve corresponding to $t = 1$.

2. (a) Let $f(x, y) = x\cos(y) - x^2 y$. Find the second partial derivative f_{xy}.
   (b) Let $f = x^2 z$ and $\mathbf{F} = (xz,\, y,\, z^2 y)$. Find ∇f (the gradient of f), $\operatorname{div}\mathbf{F}$ (the divergence of $\mathbf{F}$), and $\operatorname{curl}\mathbf{F}$ (the curl of $\mathbf{F}$).

3. (a) Find the directional derivative of the function $f(x, y, z) = xz - y$ in the direction of the vector $\vec{v} = \vec{i} - 2\vec{j} + 2\vec{k} = \langle 1, -2, 2 \rangle$ at the point $(1, 2, 0)$.
(b) Find the maximum rate of change of $f(x, y) = xy + 2\sqrt{y}$ at the point $(2, 1)$. In which direction does it occur?

4. (a) Let $z = x^3y^2 - x$. Find the equation of the tangent plane at the point $(2, 1)$.
   (b) Find equation of the tangent plane to the surface $x^2 + 2y^2 - 2z^2 = 4$ at the point $(2, -1, 1)$.

5. Find the local maximum, minimum and saddle points (if any) of the function

$$f(x, y) = 2x^2 + 4xy - y^2 + 6x - 3.$$

6. (a) Find the linear approximation for the function

$$f(x, y) = 2x^2 + y^2 + yx$$

near the point $(1, -2)$.

(b) Let $f(x, y) = xy - 2x^2y$ and $x = s - t, \quad y = s^2t$. Find the partial derivatives $\partial f/\partial s$ and $\partial f/\partial t$.

7. Find the absolute maximum and absolute minimum points of the function

$$f(x, y) = 2x^2 + 3y^2 - 4x - 3$$

on the region $0 \leq x \leq 2, \quad -1 \leq y \leq 1$. Be sure to provide the coordinates of the points and the values of absolute maximum and minimum.

8. Evaluate, by making an appropriate change of variables, the integral

$$\iint_D (x+y)\sin(x-y)\,dA$$

where D is the rectangle enclosed by the lines $x-y=0$, $x-y=4$, $x+y=1$, and $x+y=2$.

9. An agricultural sprinkler distributes water in a circular pattern over a maize field. Each hour, it supplies water to a depth of e^{-r}/r feet at a distance of $r > 0$ feet from the sprinkler. Find the total amount of water supplied per hour to the region between the circles of radius 5 and 10 feet centered at the sprinkler.

10. (a) Switch the order of integration in the iterated integral

$$\int_0^2 \left[\int_0^{x^3} f(x,y)\,dy \right] dx.$$

(b) Using a double integral, find the area of the triangle with vertices $(0,0)$, $(1,1)$, $(1,2)$.

11. Use cylindrical coordinates to find the mass of the solid that lies within both the cylinder $x^2 + y^2 = 16$ and the sphere $x^2 + y^2 + z^2 = 36$ and above the plane $z = 0$, if the material in the solid has density (mass per unit volume) given by $\rho(x, y, z) = 2$.