Instructor:_____ Name:_____

Final Exam

Calculus I; Spring 2010

Part I

Part I consists of 10 questions, each worth 5 points. Clearly show your work for each of the problems listed.

In 1-4, find y' if:

(1) $y = x^5 \cos(x)$

(2) $y = \frac{e^x}{x^2+1}$

(3) $y = (\ln(x))^{30}$

(4) $y = \sin(x^2)$

(5) Find the critical points of $y = f(x) = x(x-1)^5$

(6) Find all local/absolute maxima/minima of the function $y = 3x^4 - 6x^2$. Make sure to state both x and y values.

(7) Find all asymptotes of the function $y = \frac{2x^2+5}{x^2-9}$

(8) Find all x-values where $y = x\ln(x)$ is **decreasing**

(9) Find the most general form for the **anti-**derivative of $y = x^2(3x + 2)$

(10) **Use calculus** to find two positive numbers whose product is 6 and whose sum is minimal

Part II

Part II consists of 6 problems; the number of points for each part are indicated by [x pts]. You must show the relevant steps (as we did in class) and justify your answer to earn credit. Simplify your answer when possible.

(1) **[10 pts]** Use implicit differentiation to find the derivative y' if $\sin(xy) = x^2 + y^2$

(2) **[6 pts]** Find the linearization of the function $y = f(x) = \sqrt[3]{x}$ at $x = 8$.

(3) **[4 pts]** Use the linearization in problem **??** to estimate $\sqrt[3]{8.1}$

(4) Given the function $y = f(x) = x^3 - 3x$

(a) **[2 pts]** Find the x and y intercepts of the function.

(b) **[2 pts]** Find the open intervals where $f(x)$ is increasing and the open intervals where $f(x)$ is decreasing,

(c) **[2 pts]** Find the local maximum and local minimum values of $f(x)$. (Be sure to give the x and y coordinate of each of them).

(d) **[2 pts]** Find all open intervals where the graph of $f(x)$ is concave up and all open intervals where the graph is concave down.

(e) **[2 pts]** Find all points of inflection (be sure to give the x and y coordinate of each point).

(f) **[5 pts]** Use the above information to graph the function below. Indicate all relevant information in the graph; in particular any **x,y-intercepts, absolute/local maxima/minima and point(s) of inflection.**

(5) **[5 pts]** If $y = \frac{(x-1)^2}{(x+1)^3}$ find the absolute maximum and minimum of $f(x)$ on the interval $[0, 3]$. (Include the appropriate y values but do not simplify.)

(6) **[10 pts]** Find the dimensions of a rectangle of maximal area, located above the x-axis, if two vertices are on the x-axis and the other two are on the parabola $y = -x^2 + 6$