EGR 265, Math Tools for Engineering Problem Solving
April 13, 2009, 50 minutes

Name: ....

TEST III

Problem 1 (9+9 points)

(a) Let $f(x,y) = -x^3 + 3xy^2$. Find $f_{xx} + f_{yy}$.

(b) For the function $g(x,y) = xe^{2xy}$ find g_x, g_y and g_{xy}.

Problem 2 (9+9 points)

(a) For the function of three variables $h(x, y, z) = x\sin(yz)$ find its gradient $\nabla h(x, y, z)$.

(b) Find the directional derivative of $h(x, y, z)$ at the point $(1, 3, 0)$ in the direction of the vector $\mathbf{i} + 2\mathbf{j} - \mathbf{k}$.

Problem 3 (12+6 points + 8 bonus points)

(a) Find an equation for the tangent plane to the paraboloid $z = 2x^2 + y^2$ at the point $(1, 1, 3)$.

(b) Also, find parametric equations for the normal line of $z = 2x^2 + y^2$ at $(1, 1, 3)$.

(c) (Bonus) The curve C parameterized by $x = t$, $y = t$, $z = 3t^2$, $-\infty < t < \infty$ lies in the paraboloid $z = 2x^2 + y^2$ and goes through the point $(1, 1, 3)$ for $t = 1$. Find a tangent vector $\mathbf{v}$ to C at $(1, 1, 3)$ and a vector $\mathbf{u}$ which lies in the tangent plane of $z = 2x^2 + y^2$ at $(1, 1, 3)$ and is orthogonal to $\mathbf{v}$.

Problem 4 (12 points)

Evaluate $\int_C (2 + x^2 y)\, ds$, where C is the upper half of the unit circle $x^2 + y^2 = 1$.

Problem 5 (12 points)

Find the work done by the force field

$$F(x, y) = 6x^3 y\mathbf{i} + e^y \mathbf{j}$$

along the graph of the function $y = x^2$, $0 \leq x \leq 2$.

Problem 6 (5+5 points)

Determine for each of the following force fields if it is conservative.

(a) $F(x, y) = (x - y)\mathbf{i} + (x - 2)\mathbf{j}$

(b) $F(x, y) = (3 + 2xy)\mathbf{i} + (x^2 - 3y^2)\mathbf{j}$

Problem 7 (12 points)

For the conservative force field $F(x, y)$ from Problem 6 find a potential function $\phi(x, y)$ and calculate the work done by the force field along the curve traced by the vector function $\mathbf{r}(t) = (e^t \sin t)\mathbf{i} + (e^t \cos t)\mathbf{j}$, $0 \leq t \leq \pi$.