EGR 265, Math Tools for Engineering Problem Solving
February 2, 2009, 50 minutes

Name (Print last name first): .....

Student ID Number: ....

TEST I

Problem 1

Determine the order of the following ODEs. Also, state if they are linear or non-linear. (4P+4P+4P+4P)

(a) $y' + \cos x = y$

(b) $y' + \cos y = x$

(c) $\dfrac{y'' - \sin x}{y} = \cos x$

(d) $y''' = xyy' + x^2$

Problem 2

Which of the following functions are solutions of $2x^2y'' - xy' + y = 0$? Justify your answers! (4P+4P+4P)

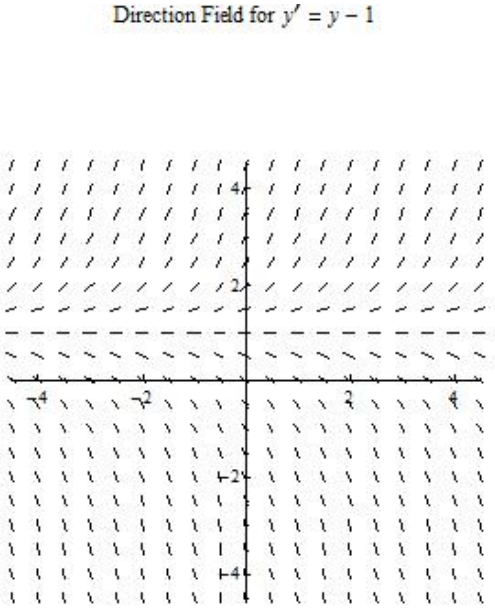
(a) $y = x$

(b) $y = x^2$

(c) $y = \sqrt{x}$

Problem 3

Below the direction field for $y' = y - 1$ is given.



(a) Sketch the solution $y(x)$ of $y' = y - 1$ with initial value $y(1) = -1$. (6P)

(b) From your sketch determine $\lim_{x\to\infty} y(x)$ and $\lim_{x\to-\infty} y(x)$. (4P)

Problem 4

Solve the IVP (14P)

$$e^{-x}y' = y^2, \quad y(0) = -\frac{1}{2}$$

Problem 5

Solve the IVP (14P)

$$\frac{y' + y}{x} = 1, \quad y(0) = 1$$

Problem 6

Solve the IVP (14P)

$$y' = xy\sqrt{x^2+1}, \quad y(0) = 1$$

Problem 7

The mass of a radioactive material is given by $m(t)$, where the time t is measured in years and the mass in grams. An initial mass of $m(0) = 10$ grams decays at a constant rate $k = m'(t)/m(t)$. After one year 8 grams of the material are left.

(a) Find the decay rate k by solving the differential equation for $m(t)$. (10P)

(b) Find an expression for the time t_h at which only 5 grams of the material are left (t_h is the so-called *half-life* of the material). (4P)

Note: Your answers will contain natural logarithms which do not need to be evaluated.

(a)

(b)