MA 125 CALCULUS I SPRING 2009
Friday, May 01, 2009

FINAL EXAM

Name (Print last name first):..

Student Signature: ....

Instructor: .... Section: ....

PART I

Part I consists of 10 questions. Place your answer on the answer-line next to the question. Space is provided between questions for you to work each question (if you wish). No partial credit is awarded on Part I problems, only your entry on the answer line will be graded.

Each question is worth 4 points.

Question 1

Evaluate $\lim_{x \to 3} \dfrac{x^2 - 7x + 12}{x - 3}$. Answer: ....

Question 2

Evaluate $\lim_{x \to 0} \dfrac{e^x + \cos(x) - 2}{x}$. Answer: ....

Question 3

Find y' if $y = \dfrac{x^3}{x+1}$.

Answer: ....

Question 4

Find the value of x for which the curve $y = (x+3)e^x$ has a horizontal tangent line.

Answer: ....

Question 5

Find an equation of the tangent line to the curve $y = 3 + x\ln(x+1)$ at the point $(0,3)$.

Answer: ....

Question 6

Find y' if $y = \sqrt[5]{x^5 + 3x^2 + 1}$.

Answer: ....

Question 7

Find all the critical numbers of the function $f(x) = \frac{4}{7}\,x^{7/4} - 4x^{1/4}$.

Answer: ....

Question 8

Find the open interval(s) on which the function $f(t) = \dfrac{e^t}{t+1}$ is increasing.

Answer: ....

Question 9

Find all inflection points of the curve $y = 2x^6 - 5x^4 + 3$. [Be sure to give the x and the y coordinates of each point!]

Answer: ....

Question 10

Find the most general antiderivative of $f(x) = 1 + \sqrt{x} + \cos(x)$.

Answer: ....

PART II

Each problem is worth 10 points.

Part II consists of 6 problems. You must show the relevant work on this part of the test to get full credit; that is, your solution must include enough detail to justify any conclusions you reach in answering the question. Partial credit may be awarded on Part II problems where it is warranted.

Problem 1

Consider the equation

$$x^2 + xy + y^2 - 3 = 0$$

in which y is implicitly defined as a function of x.

(a) Use implicit differentiation to find y'.

(b) Is the curve $x^2 + xy + y^2 - 3 = 0$ rising or falling at the point $(1, 1)$? (Justify your answer!)

(c) Find an equation of the tangent line to the curve $x^2 + xy + y^2 - 3 = 0$ at the point $(1, 1)$.

Problem 2

Consider the function

$$f(x) = 3x^2 - x^3 - 1.$$

(a) Find the open interval(s) where $f(x)$ is increasing, and the open interval(s) where it is decreasing.

(b) Find all local maximum and minimum points of $f(x)$. [Be sure to give the x and the y coordinate of each point!]

(c) Find the open interval(s) where $f(x)$ is concave down, and the open interval(s) where it is concave up.

Problem 2 (continued)

(d) Find the inflection point(s) of $f(x)$. [Be sure to give the x and the y coordinate!]

(e) Sketch a graph of $f(x) = 3x^2 - x^3 - 1$. (Clearly indicating the relevant items above on your graph.)

Problem 3

Consider the function

$$f(x) = \sqrt[4]{1-x}.$$

(a) Find the linearization (or linear approximation) of $f(x)$ at $a = 0$.

(b) Use the linearization of $f(x)$ to find an approximation of $\sqrt[4]{0.9}$. (Show your answer to 3 decimal places!)

(c) Another way to find an approximation of $\sqrt[4]{0.9}$ is to use Newton's method to find a root of the equation $x^4 - 0.9 = 0$.
Use Newton's method with initial approximation $x_1 = 1$ to find x_2, the second approximation to the root of the equation

$$x^4 - 0.9 = 0.$$

Problem 4

A tiny arrow is shot straight upward from the ground at time $t = 0$. It is known that its height (in meters) after t seconds is given by

$$s(t) = 40t - 5t^2.$$

Answer the following questions.

(a) Find the velocity $v(t)$ of the arrow after t seconds.

(b) In which direction is the arrow moving after 5 seconds of travel? (Hint: Your answer should be either "upward" or "downward," and you must justify your answer!)

(c) Find the acceleration $a(t)$ of the arrow after t seconds.

(d) What is the maximum height the arrow will reach? (You must justify your answer!)

(e) How many seconds will elapse before the arrow strikes the ground again? And, determine the impact velocity. (You must justify your answers!)

Problem 5

The volume of an open top (cylindrical) barrel with circular base is given by $V = \pi r^2 h$, where r is the radius of the circular base and h is the height of the barrel. The surface area of the (open top) barrel is given by $S = \pi r^2 + 2\pi r h$. Assume that $S = 27\ \mathrm{m}^2$ of material is available to make the barrel. Find the dimensions of this barrel that will yield the maximum volume.

(Hint: In your calculations, you should express the volume as a function of r only.)

Problem 6

This problem has two separate questions. Answer each question!

(a) Find the position $s(t)$ of an object moving in a straight line if it is known that its acceleration is given by $s''(t) = 4 - 6t - 12t^2$ km/s^2, and that its initial position $s(0) = 2$ km and its initial velocity $s'(0) = 1$ km/s.

(b) Find the antiderivative $F(x)$ of the function

$$f(x) = \sec^2(x) + 1$$

on the interval $-\pi/2 < x < \pi/2$, given that

$$F(\pi/4) = \pi/4.$$

Summary of scores on problems - for grading purposes only. Do not enter any problem solutions or work on this page.

	Points
Part I	
Part II - Problem 1	
Problem 2	
Problem 3	
Problem 4	
Problem 5	
Problem 6	
Total Test Score	