SPRING 2008 — MA 227 — FINAL

Name: ________________________________

1. Part I

There are 10 problems in Part I, each worth 4 points. Place your answer on the line below the question. In Part I, there is no need to show your work, since only your answer on the answer line will be graded.

(1) Compute the cross product of the vectors $\langle 1, 1, 0\rangle$ and $\langle 2, 1, -2\rangle$.

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(2) Find the gradient of the function $f(x, y) = x^2 + 2y^2$.

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(3) Find the directional derivative of the function $h(x, y) = xy$ at the point $(1, 1)$ in the direction of the vector $\langle 1, 1\rangle$.

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(4) Find a parametrization for a circle with radius 2 centered at the origin.

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(5) Find an equation of the plane with normal $\langle 1, 1, 0\rangle$ which contains the origin.

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(6) Find the linearization $L(x, y)$ of $f(x, y) = xe^y$ at the point $(1, 0)$.

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(7) Evaluate $\iint_D 2dA$ where D is the unit disc with radius 1.

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(8) Find a function f such that $\nabla f = \langle 2y^{3/2}, 3xy^{1/2} \rangle$.

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(9) Evaluate the line integral $\int_C 2ds$ when C is the straight line from $(0, 0)$ to $(0, 1)$.

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(10) Compute $\operatorname{curl} \mathbf{F}$ when $\mathbf{F}(x, y, z) = \langle xy, \cos(xy), 0 \rangle$.

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2. Part II

There are 6 problems in Part II, each worth 10 points. On Part II problems show all your work! Your work, as well as the answer, will be graded. Your solution must include enough detail to justify any conclusions you reach in answering the question.

(1) A ball is thrown horizontally (angle $\alpha = 0$) from a tower of height $5m$ with initial speed of $1m/s$.
 (a) Find the vector equation for acceleration, velocity, and position.
 (b) How far away from the tower does the ball touch the ground?
 Use $g = 10m/s^2$.

(2) Find the length of the arc of the circular helix with vector equation $r(t) = \langle \cos t, \sin t, t \rangle$ from the point $(1, 0, 0)$ to the point $(1, 0, 2\pi)$.

(3) Use Lagrange multipliers to find the maximum and minimum values of the function $f(x, y) = x + 2y$ subject to the constraint $x^2 + 4y^2 = 2$.

(4) Let C be the curve consisting of the sides of the triangle with vertices $(0,0)$, $(0,1)$, and $(0,1)$. Evaluate $\int_C (xydx + x^2dy)$ by using Green's Theorem as well as by direct integration.

(5) A surface S is given parametrically by $\mathbf{r}(u,v) = \langle u+v, u-v, 1-u\rangle$ where $u^2+v^2 \leq 1$ and $v \geq 0$. Evaluate the area of the surface S.

(6) Use spherical coordinates to evaluate the triple integral

$$\iiint_E z\,dV$$

where E is the following solid which lies in the first octant:

$$E = \{1 \leq x^2 + y^2 + z^2 \leq 4, \quad x \geq 0, y \geq 0, z \geq 0\}.$$