MA 126-6D, CALCULUS II
April 21, 2008

Name (Print last name first): .....

Student Signature: .....

TEST IV

No calculators are permitted!

PART I - Basic Skills

Each question is worth 5 points.

Part I consists of 6 questions. Clearly write your answer (only) in the space provided after each question. You need not show your work for this part of the test. No partial credit is awarded for this part of the test!

Question 1

Find the length of the "arc" $y = 3 + 2\,x$, when $0 \leq x \leq 1$. (Your answer must be a real number!)

Answer: .....

Question 2

Find the limit of the **sequence** given by $a_n = \cos\left(\frac{5}{n}\right)$. (Your answer must be a number!)

Answer: .....

Question 3

Determine whether the **geometric series** $\sum_{n=1}^{\infty} \left(\frac{7}{2}\right)^{n-1}$ is convergent or divergent. If it is convergent, find its sum.

Answer: .....

Question 4

Determine whether the infinite **series** $\sum_{n=1}^{\infty} \frac{n+1}{2n-1}$ is convergent or divergent.

Answer: .....

Question 5

Determine whether the infinite **p-series** $\sum_{n=1}^{\infty} \frac{1}{n^6}$ is convergent or divergent.

Answer: .....

Question 6

Determine whether the **alternating series** $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3}$ is divergent, absolutely convergent, or conditionally convergent.

Answer: .....

PART II - Problem Solving skills

Each problem is worth 14 points.

Part II consists of 5 problems. You must show your work on this part of the test to get full credit. Displaying only the final answer (even if correct) without the relevant steps will not get full credit.

Problem 1

(a) Find **the length of the curve**

$$y = 1 + 2x^{3/2}$$

when $0 \leq x \leq 1$.

(Hint: Integration by substitution might prove useful here!)

(b) Find **the length of the curve** given by the parametric equations $x = 3\cos t$, $y = 3\sin t$ for $0 \leq t \leq \pi/2$.

(Hint: It might be helpful to notice that this arc is a portion of a well known graph in the plane!)

Problem 2

(a) Find the limit of the convergent **sequence** defined by

$$a_1 = 1, \qquad a_{n+1} = 3 - \frac{1}{a_n}.$$

(b) Find **the values of** x for which the **geometric series**

$$\sum_{n=1}^{\infty} \left(\frac{x+2}{5} \right)^{n-1}$$

converges? Write your answer in interval notation!

Problem 3

(a) Find the numerical value of c for which

$$\sum_{n=1}^{\infty} \frac{1}{(3+c)^n} = 5.$$

(Hint: Note that the series starts from $n = 1$.)

(b) Use the **Integral Test** to determine whether the **series**

$$\sum_{n=3}^{\infty} \frac{3}{n(\ln n)^5}.$$

is convergent or divergent. (Show your work!)

Problem 4

(a) Consider the **series**

$$\sum_{n=1}^{\infty} \frac{n^2}{n^4+1}.$$

Determine whether the series is convergent or divergent. (You must justify your answer!)

(b) Consider the **alternating series**

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}.$$

Determine whether the series is absolutely convergent, conditionally convergent, or divergent. (Only one of these three choices will be accepted as an answer, and you must justify your choice!)

Problem 5

Consider the **series**

$$\sum_{n=1}^{\infty} \frac{(-5)^n}{\sqrt{n}}.$$

Answer all the following questions.

(a) Determine whether the ratio test is conclusive or inconclusive. (Justify your answer!)

(b) Determine whether the series is absolutely convergent, conditionally convergent, or divergent. (Only one of these three choices will be accepted as an answer, and you must justify your choice!)

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