MA 126-6D,   CALCULUS II
March 31, 2008

Name (Print last name first): ....

Student Signature: ....

$$\boxed{\text{TEST III}}$$

$$\boxed{\text{No calculators are permitted!}}$$

$$\boxed{\text{PART I - Basic Skills}}$$

Each question is worth 5 points.

Part I consists of 6 questions. Clearly write your answer (only) in the space provided after each question. You do not need not to show your work for this part of the test. No partial credit is awarded for this part of the test!

Question 1

Evaluate the indefinite integral $\displaystyle\int \frac{x}{x-6}\,dx.$

Answer: ....

Question 2

Evaluate the indefinite integral $\displaystyle\int 3x^2 \sin(x^3),dx.$

Answer: ....

Question 3

Determine whether the improper integral is **convergent** or **divergent**. **Evaluate the integral if it is convergent**.

$$\int_{1}^{\infty} \frac{1}{\sqrt{x}} \, dx$$

Answer: ....

Question 4

Determine whether the improper integral is **convergent** or **divergent**. **Evaluate the integral if it is convergent**.

$$\int_{0}^{\infty} \frac{2x}{(x^2 + 2)^2} \, dx$$

Answer: ....

Question 5

Find the area of the region bounded by the parabola $y = x^2 + 1$, the horizontal line $y = 0$, and the vertical lines $x = 0$ and $x = 1$.

Answer: ....

Question 6

Find the volume of the solid obtained by rotating the curve $y = x^3$ about the y-axis for $0 \le y \le 1$.

Answer: ....

PART II - Problem Solving skills

Each problem is worth 14 points.

Part II consists of 5 problems. You must show your work on this part of the test to get full credit. Displaying only the final answer (even if correct) without the relevant steps will not get full credit.

Problem 1

(a) Evaluate the definite integral

$$\int_0^{\pi/2} \sin^5 x \cos x \, dx.$$

(b) Evaluate the indefinite integral

$$\int \frac{x-1}{x^2 + 3x + 2} \, dx.$$

Problem 2

(a) Evaluate the indefinite integral
$$\int \frac{x-2}{x^3+x}\, dx.$$

(b) Make a substitution to express the integrand as a rational function and then evaluate the integral

$$\int_9^{16} \frac{\sqrt{x}}{x-4}\, dx \qquad (\text{Set } u = \sqrt{x}, \text{ and note that } x = u^2.)$$

Problem 3

(a) Determine whether the (improper) integral

$$\int_e^\infty \frac{1}{x(\ln x)^3} \, dx$$

is **convergent** or **divergent**. **Evaluate the integral if it is convergent**.

(b) Find the exact area of the region between the graph of the function $f(x) = 4x^3 e^{-x^4}$ and the x-axis when $0 \le x < \infty$.

Problem 4

(a) Find the **area** of the region **enclosed** by the parabola $y = x^2$ and the line $y = 2 - x$. (Hint: Sketching the region might prove useful here!)

(b) Find the numerical values of c such that the **area** of the region **bounded** by the parabolas $y = x^2 - c^2$ and $y = c^2 - x^2$ is equal to $\frac{64}{3}$. (Hint: You should find two values in all!)

Problem 5

(a) Find the **volume** of the solid obtained by rotating about the x-**axis** the region **bounded** by the curve $y = \sqrt{x}$, the horizontal line $y = 0$ and the vertical lines $x = 0$ and $x = 2$.

(b) Find the **volume** of the solid obtained by rotating about the x-**axis** the region **bounded** by the curve $y = \frac{1}{x}$, the horizontal line $y = 0$ and the vertical lines $x = 1$ and $x = 3$.

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