

MA 125-8B, CALCULUS I

January 31, 2008

Name (Print last name first):

Student ID Number:

TEST I

PART I

Part I consists of 7 questions. Clearly write your answer (only) in the space provided after each question. You do not need not to show your work for this part of the test. No partial credit is awarded for this part of the test!

Each question is worth 5 points.

Question 1Given that $\lim_{x \rightarrow a} f(x) = 5$ and $\lim_{x \rightarrow a} h(x) = -2$, find

$$\lim_{x \rightarrow a} \frac{3h(x)}{2h(x) + f(x)}.$$

Question 2If $-2x \leq g(x) \leq -x^2 - 2x + 1$ for all $x > 1$, evaluate

$$\lim_{x \rightarrow 1^+} g(x)$$

if the limit exists.

Question 3

Evaluate the limit

$$\lim_{x \rightarrow 2\pi} \cos \left(3\pi \cos \left(\frac{x}{2} \right) \right).$$

Question 4

Find the limit

$$\lim_{x \rightarrow \infty} \frac{-x^5 + 4x^2}{8x^5 - x^3 + 7}.$$

Question 5

Given that $g(2) = -5$ and $g'(2) = 4$, find an equation of the tangent line to the graph of $y = g(x)$ at $x = 2$.

Question 6

Where is the function $f(x) = 2|x - 7|$ **not** differentiable?

Question 7

Calculate the limit $\lim_{x \rightarrow \infty} \tan^{-1}(x - x^3)$.

PART II

Each problem is worth 14 points.

Part II consists of 5 problems. You must show your work on this part of the test to get full credit. Displaying only the final answer (even if correct) without the relevant steps will not get full credit.

Problem 1

Consider the function

$$f(x) = \begin{cases} -x + 5 & \text{for } x < 2, \\ 4x - 1 & \text{for } x \geq 2. \end{cases}$$

(a) Evaluate

$$\lim_{x \rightarrow 2^-} f(x).$$

(b) Evaluate

$$\lim_{x \rightarrow 2^+} f(x).$$

(c) Is this function continuous at $x = 2$? (Justify your answer!)

Problem 2

Sketch the graph of an example of a function f such that

$$\begin{aligned} \lim_{x \rightarrow 0^-} f(x) &= -2, & \lim_{x \rightarrow 0^+} f(x) &= 1, & f(0) &= 2, \\ \lim_{x \rightarrow 2^-} f(x) &= 0, & \lim_{x \rightarrow 2^+} f(x) &= -1, & f(2) &= -1. \end{aligned}$$

Problem 3

(a) Evaluate the limit

$$\lim_{x \rightarrow 3} \frac{x^2 - 2x - 3}{x^2 - 7x + 12}.$$

(b) Evaluate the limit

$$\lim_{x \rightarrow 9} \frac{6 - \sqrt{x}}{\sqrt{1+x}}.$$

Problem 4

- (a) For what (numerical) value of the constant c is the function

$$f(x) = \begin{cases} -x^2 + cx & \text{if } x > 1 \\ 3cx^3 + 1 & \text{if } x \leq 1 \end{cases}$$

continuous on $(-\infty, \infty)$? (Justify your answer!)

- (b) The displacement (in meters) of a particle moving in a straight line is given by the equation of motion $s = -7t + 30$, where t is measured in seconds. Find the velocity of the particle at time $t = 9$ seconds.

Problem 5

Let $F(x) = \frac{7}{x}$.

(a) Use the limit definition of the derivative to find $F'(2)$.

(b) Find an equation of the tangent line to the curve $y = \frac{7}{x}$ at the point $(2, 3.5)$.