Final Exam

Calculus III

April 27, 2007

Time : 2 hr 30 min

Instructions

Calclulators are **not** permitted. You may use your textbook for reference, but no notes. Also you may use the back of the pages if the space below is not sufficient; however, clearly indicate where the relevant work for a problem is located if not directly above the question. Show your work as correct answers without justification will earn no credit.

Do NOT write in this box

Part I	
Question 11	
Question 12	
Question 13	
Question 14	
Question 15	
Question 16	
Total	

Part I

In this section there are 10 questions worth 4 points each for a total of 40 points. Work each problem in the space allotted and place your answers on the lines provided on the right.

Question 1. Find the rate of change of $f(x, y, z) = e^{xy} + \ln(xz)$ per unit distance at $P(0, 0, \frac{1}{2})$ in the direction of $Q(2, 2, \frac{5}{2})$.

Answer: $\frac{7}{3}$

Question 2. Find parametric equations for the tangent line to the curve given by $\vec{r}(t) = \langle t, t^3, \ln(t) \rangle$ at the point where $t = 1$.

Answer: $x = 1 + t,\ y = 1 + 3t,\ z = t$

Question 3. Express $\int_0^2 \int_0^{2x} f(x,y)\,dydx$ as an equivalent iterated integral with the order of integration reversed.

Answer: $\int_0^4 \int_{\frac{y}{2}}^2 f(x,y)\,dydx$

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Question 4. Suppose the radius of a right circular cone is increasing at the rate of 2 in/sec while the height is decreasing by 1 in/sec. Use a chain rule to find the rate the volume is changing when the radius is 10 in. and the height is 5 in.

Answer: $\frac{100\pi}{3}$

Question 5. Find the divergence of the vector field

$$F = \frac{x}{x^2+y^2+z^2}\vec{i} + \frac{y}{x^2+y^2+z^2}\vec{j} + \frac{z}{x^2+y^2+z^2}\vec{k}$$

Answer: $\dfrac{1}{x^2+y^2+z^2}$

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Question 6. Find the work done by the force field $\vec{F}(x,y) = x\sin(y)\vec{i} + y\vec{j}$ on a particle that moves along the parabola $y = x^2$ from $(-1,1)$ to $(0,0)$.

Answer: $\dfrac{1}{2}(\cos(1)-1)$

Question 7. Find the curvature of the graph of $y = \tan(x)$ at the point $\left(\frac{\pi}{4}, 1\right)$

Answer: $\frac{4}{5\sqrt{5}}$

Question 8. Find the total differential df if $f(x, y) = \tan^{-1}\left(\frac{x}{y}\right)$

Answer: $df = \frac{y}{x^2 + y^2}dx - \frac{x}{x^2 + y^2}dy$

Question 9. Evaluate the line integral of $f(x, y) = 3 - x - y$ along the helix $\vec{r} = (\cos 4t)\vec{i} + (\sin 4t)\vec{j} + (t)\vec{k}$, $0 \leq t \leq 2\pi$.

Answer: $6\sqrt{17}\pi$

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Question 10. If $\vec{F}(x, y) = -(xy \sin xy - \cos xy)\vec{i} - (x^2 \sin xy)\vec{j}$, find a function f such that $\vec{F} = \nabla f$.

Answer: $f(x, y) = x \cos xy$

Part II

This section consists of 6 problems worth 10 points each for a total of 60 points. Work each problem in the space allotted.

Question 11. Determine the equation of the level surface of the function $f(x, y, z) = -x + \dfrac{y^2}{4} + z^2$ that goes through the point $P(-3, 4, 2)$. Also, find the equations of the tangent plane and the normal line to this level surface at P.

Answer:

- Level Surface: $-x + \dfrac{y^2}{4} + z^2 = 11$
- Tangent Plane: $-x + 2y + 4z = 19$
- Normal Line: $x = -3 - t$, $y = 4 + 2t$, $z = 2 + 4t$

Question 12. Use Lagrange multipliers to find the maximum and minimum values of the function

$$f(x, y, z) = 16x - 8z$$

subject to the constraint

$$x^2 + 15y^2 + z^2 = 20$$

Answer:

- $f_{max} = 80$
- $f_{min} = -80$

Question 13. Find the critical points of $f(x, y) = 8x^3 - 24xy + y^3$ and determine whether each is a local minimum, local maximum or neither.

Answer:

- Local min. points: $(2, 4)$
- Local max. points: None
- Local min. points: $(0, 0)$

Question 14. Find the volume of the solids bounded by the paraboloids $z = 3x^2 + y^2$ and $z = 12 - x^2 - 3y^2$.

Answer:

- Volume $= 18\pi$

Question 15. Let C be the positively oriented boundary of the triangle with vertices $(0,0)$, $(1,2)$ and $(0,2)$. Calculate

$$\oint 4x^2ydx + 2ydy$$

(a) by direct integration using line integrals and
(b) by Green's Theorem.

Answer:

- $\oint 4x^2ydx + 2ydy = -\frac{2}{3}$

Question 16. Let $\iiint_E x\ dV$ be a triple integral where E is the solid tetrahedron with vertices $(0,0,0)$, $(1,0,0)$, $(0,2,0)$ and $(0,0,3)$.

Express dV appropriately and determine the corresponding six limits of integration needed if one were to evaluate the integral. (*However, do not actually evaluate the integral. Just set it up for evaluation.*)

Answer:

- $\iiint_E x\ dV = \int_0^1 \int_0^{-2x+2} \int_0^{3-3x-\frac{3}{2}y} x\, dz dy dx$