Final Exam

Calculus III April 27, 2007

Time: 2 hr 30 min

Instructions

Calculators are **not** permitted. You may use your textbook for reference, but no notes. Also you may use the back of the pages if the space below is not sufficient; however, clearly indicate where the relevant work for a problem is located if not directly above the question. Show your work as correct answers without justification will earn no credit.

Do NOT write in this box		
Part I		
Question 11		
Question 12		
Question 13		
Question 14		
Question 15		
Question 16		
Total		

Part I

In this section there are 10 questions worth 4 points each for a total of 40 points. Work each problem in the space allotted and place your answers on the lines provided on the right.

Question 1. Find the rate of change of $f(x, y, z) = e^{xy} + \ln(xz)$ per unit distance at $P(0, 0, \frac{1}{2})$ in the direction of $Q(2, 2, \frac{5}{2})$.



Question 2. Find parametric equations for the tangent line to the curve given by $\vec{r}(t) = \langle t, t^3, \ln(t) \rangle$ at the point where t = 1.

Answer: x = 1 + t, y = 1 + 3t, z = t

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Question 3. Express $\int_0^2 \int_0^{2x} f(x, y) \, dy dx$ as an equivalent iterated integral with the order of integration reversed.

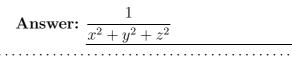
Answer: $\int_0^4 \int_{\frac{y}{2}}^2 f(x,y) \, dy dx$

Question 4. Suppose the radius of a right circular cone is increasing at the rate of 2 in/sec while the height is decreasing by 1 in/sec. Use a chain rule to find the rate the volume is changing when the radius is 10 in. and the height is 5 in.

Answer:	100π
	3

Question 5. Find the divergence of the vector field

$$F = \frac{x}{x^2 + y^2 + z^2}\vec{i} + \frac{y}{x^2 + y^2 + z^2}\vec{j} + \frac{z}{x^2 + y^2 + z^2}\vec{k}$$

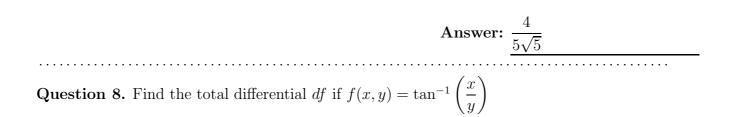


Question 6. Find the work done by the force field $\vec{F}(x,y) = x \sin(y)\vec{i} + y\vec{j}$ on a particle that moves along the parabola $y = x^2$ from (-1,1) to (0,0).

.

Answer: $\frac{1}{2}(\cos(1) - 1)$

Question 7. Find the curvature of the graph of $y = \tan(x)$ at the point $\left(\frac{\pi}{4}, 1\right)$



Answer:
$$df = \frac{y}{x^2 + y^2} dx - \frac{x}{x^2 + y^2} dy$$

Question 9. Evaluate the line integral of f(x,y) = 3 - x - y along the helix $\vec{r} = (\cos 4t)\vec{i} + (\sin 4t)\vec{j} + (t)\vec{k}, 0 \le t \le 2\pi$.

Answer: $6\sqrt{17}\pi$

Question 10. If $\vec{F}(x,y) = -(xy\sin xy - \cos xy)\vec{i} - (x^2\sin xy)\vec{j}$, find a function f such that $\vec{F} = \nabla f$.

.

Answer: $f(x, y) = x \cos xy$

Part II

This section consists of 6 problems worth 10 points each for a total of 60 points. Work each problem in the space allotted.

Question 11. Determine the equation of the level surface of the function $f(x, y, z) = -x + \frac{y^2}{4} + z^2$ that goes through the point P(-3, 4, 2). Also, find the point of the function of the function of the point P(-3, 4, 2). that goes through the point P(-3, 4, 2). Also, find the equations of the tangent plane and the normal line to this level surface at P.

Answer:

- Level Surface: $-x + \frac{y^2}{4} + z^2 = 11$ Tangent Plane: -x + 2y + 4z = 19
- Normal Line: x = -3 t, y = 4 + 2t, z = 2 + 4t

Question 12. Use Lagrange multipliers to find the maximum and minimum values of the function f(x,y,z) = 16x - 8z

subject to the constraint

$$x^2 + 15y^2 + z^2 = 20$$

Answer:

•
$$f_{max} = 80$$

•
$$f_{min} = -80$$

Answer:

- Local min. points: (2,4)
- Local max. points: None
- Local min. points: (0,0)

SECTION:

Question 14. Find the volume of the solids bounded by the paraboloids $z = 3x^2 + y^2$ and $z = 12 - x^2 - 3y^2$.

Answer: • Volume = 18π

$$\oint 4x^2ydx + 2ydy$$

- (a) by direct integration using line integrals and
- (b) by Green's Theorem.

Answer:

• $\oint 4x^2ydx + 2ydy = -\frac{2}{3}$

Question 16. Let $\iiint_E x \, dV$ be a triple integral where E is the solid tetrahedron with vertices (0,0,0), (1,0,0), (0,2,0) and (0,0,3).

Express dV appropriately and determine the corresponding six limits of integration needed if one were to evaluate the integral. (However, do not actually evaluate the integral. Just set it up for evaluation.)

Answer:

•
$$\iiint_E x \ dV = \int_0^1 \int_0^{-2x+2} \int_0^{3-3x-\frac{3}{2}y} x \ dz \ dy \ dx$$