

STUDENT NAME: \_\_\_\_\_

## Test 4

Calculus III - MA 227 7B

April 25, 2007

*Time* : 60 min

### Instructions

This test consists of two parts.

There are 5 questions in this part, each worth 4 points, for a total of 20 points.

In Part II, there are 3 questions, each worth 10 points, for a total of 30 points.

You may use the back of the pages if the space next to questions is insufficient. The back pages may also be used for scratch work. However clearly indicate so if you are using it for scratch. **Leave all test sheets stapled.**

*Do NOT write in this box*

Part I	_____
Question 6	_____
Question 7	_____
Question 8	_____
<b>Total</b>	_____

STUDENT NAME: \_\_\_\_\_

## Part I

Write your answers in the place indicated

**Question 1.** Find the Jacobian of the transformation

$$x = \frac{u}{u+v} \quad y = \frac{v}{u+v}$$

.....

**Question 2.** Find the gradient vector field of  $f$

$$f(x, y, z) = x \cos\left(\frac{y}{z}\right)$$

STUDENT NAME: \_\_\_\_\_

**Question 3.** Evaluate  $\int_C x^2 z \, ds$ ,  $C$  is the line segment from  $(0, 6, -1)$  to  $(4, 1, 5)$ .

.....  
**Question 4.** Find a function  $f$  such that  $\vec{F} = \nabla f$ .

$$\vec{F}(x, y) = (1 + 2xy + \ln x)\vec{i} + x^2\vec{j}$$

STUDENT NAME: \_\_\_\_\_

**Question 5.** Use Green's Theorem to evaluate the line integral along the given positively oriented curve.

$$\int_C x^2y^2 dx + 4xy^3 dy$$

where  $C$  is the triangle with vertices  $(0, 0)$ ,  $(1, 3)$  and  $(0, 3)$ .

STUDENT NAME: \_\_\_\_\_

## Part II

**Question 6.** Evaluate

$$\iiint_H (x^2 + y^2) dV$$

where  $H$  is the right hemisphere ( $y \geq 0$ ) of  $x^2 + y^2 + z^2 = 9$

**(10 points)**

STUDENT NAME: \_\_\_\_\_

**Question 7.** Evaluate the integral

$$\iint_R (x^2 - y^2)e^{x+y} dA$$

where  $R$  is the rectangle enclosed by the lines  $x - y = 0$ ,  $x - y = 7$ ,  $x + y = 0$ , and  $x + y = 3$ , by making an appropriate change of variables.

**(10 points)**

STUDENT NAME: \_\_\_\_\_

**Question 8.** A particle starts at the point  $(0, 0)$ , moves along the  $X$ -axis to  $(6, 0)$ , and then along the semicircle  $y = \sqrt{36 - x^2}$  to the point  $(0, 6)$  and then back to the starting point along the  $Y$ -axis. Find the work done on this particle by the force field

$$F(x, y) = \langle 3x, x^3 + 3xy^2 \rangle$$

**(10 points)**