SPRING 2006 — MA 227-6D — TEST 2

Name: _____

1. Part I

There are 5 problems in Part I, each worth 4 points. Place your answer on the line below the question. In Part I, there is no need to show your work, since only your answer on the answer line will be graded.

(1) Evaluate $g(3, -6, 5)$ if $g(x, y, z) = \ln(76 - x^2 - y^2 - z^2)$.

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(2) Find the first partial derivatives of $f(x, y) = (x - 3y)/(x + 3y)$.

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(3) Find the linearization $L(x, y)$ of $f(x, y) = x\sqrt{y}$ at the point $\langle 1, 16\rangle$.

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(4) Find the gradient of $f(x, y, z) = x^4 e^{yz}$.

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(5) Use the chain rule to find the partial derivative z_t, if $z(t) = f(x(t), y(t))$, $f(x, y) = xy$, and if $x(t)$ and $y(t)$ are differentiable.

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2. Part II

There are 2 problems in Part II, each worth 10 points. On Part II problems partial credit is awarded where appropriate. Your solution must include enough detail to justify any conclusions you reach in answering the question.

(1) Use Lagrange multipliers to find the maximum and minimum values of $f(x, y) = xy$ on the ellipse given by $2x^2 + 8y^2 = 4$. Where do they occur?

(2) Suppose that over a certain region of space the electrical potential V is given by

$$V(x, y, z) = 5x^2 - 4xy + xyz.$$

(a) Find the rate of change of the potential at the point $P = \langle 2, 5, 3 \rangle$ in the direction of the vector $\mathbf{i} + \mathbf{j} - \mathbf{k}$.
(b) Find the rate of change of the potential at the point P in the direction towards the point $Q = \langle 1, 3, 5 \rangle$.
(c) In which direction does V change most rapidly at P?
(d) What is the maximum rate of change of V at P?