CALCULUS I

March 23, 2006

Name (Print last name first): ....

Student ID Number: ....

TEST III

PART I

Part I consists of 7 questions. Clearly write and circle your answer (only) in the space provided after each question. Do not show your work for this part of the test. No partial credit is awarded on Part I problems.

Each question is worth 4 points.

Question 1

Find the *slope of the tangent line* to the circle $x^2 + y^2 = 25$ at the point $(-3, 4)$.

Question 2

Find $f'(x)$ when $f(x) = \sin^{-1} x$.

Question 3

Find the differential dy when $y = \ln(x^2 + 1)$.

Question 4

Let $f(x) = (\ln x)g(x)$, where $g(1) = -3$ and $g(x)$ is differentiable at $x = 1$. Find the numerical value of $f'(1)$.

Question 5

Does the function $f(x) = x^{2/3}$ have a critical number? [Your answer must be either yes or no!]

Question 6

Find the critical number of the function $f(x) = 5x^2 + 4x$.

Question 7

Find the derivative of the function $f(x) = \ln\left(\dfrac{x+1}{x-1}\right)$. [Hint: Properties of the logarithmic function might prove useful here!]

PART II

Each problem is worth 8 points.

Part II consists of 9 problems. You must show your relevant work on this part of the test to get full credit. Displaying only the final answer (even if correct) without the relevant step(s) will not get full credit.

Problem 1

Find the linearization (or tangent line approximation) of the function

$$f(x) = \sqrt{5-x}$$

at the number $a = 1$.

Problem 2

Consider the curve given by the equation

$$y + x \sin y = x^2$$

in which y is implicitly defined as a function of x.

(a) Use implicit differentiation to find the derivative y'.

(b) Find the slope of the tangent line to the above curve at the point $(0, 0)$.

Problem 3

Consider the function

$$f(x) = x^3 - 3x + 1.$$

(a) Find, if any, all critical numbers of the function $y = f(x)$ which are inside the open interval $(-1, 2)$.

(b) Find the *absolute minimum value* of the function $y = f(x)$ on the *closed* interval $[-1, 2]$. (Hint: Check end-points as well!)

(c) Find the *absolute maximum value* of the function $y = f(x)$ on the *closed* interval $[-1, 2]$.

Problem 4

Use logarithmic differentiation to find the derivative of the function

$$y = x^x.$$

Problem 5

The edge of a cubic box was found to be 3 ft with a possible error in measurement of 0.01 ft.

Use *differentials* to estimate

(a) the maximum *possible error* in computing the volume of the box.

(b) the maximum possible *relative error* in computing the volume of the box.

(c) the maximum possible *percentage error* in computing the volume of the box.

[Hint: The volume of a cube is given by $V(x) = x^3$ where x is the length of each edge of a cube.]

Problem 6

Use Newton's method with $x_1 = 1$ to find x_2, the second approximation to the root of the equation

$$x - 1 + \tan^{-1} x = 0.$$

$\left[\text{Hint: } \tan^{-1} 1 = \frac{\pi}{4}\right].$

Problem 7

A gun-boat fires a flare straight up into the air from the sea-level with an initial velocity of 64 ft/sec. It is know that the height of the flare at each time t is given by

$$h(t) = 64t - 16t^2.$$

(a) Find the velocity of the flare.

(b) Find the maximum height that the flare will reach.

Problem 8

Consider the function

$$f(x) = 2x^3 + 3x^2 - 12x + 1.$$

(a) Find the derivative $f'(x)$.

(b) Find the critical numbers of the function $y = f(x)$.

Problem 9

A ladder 5 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 2 ft/sec, how fast is the top of the ladder sliding down when the bottom of the ladder is 3 ft from the wall?

(picture not included)