MA 126: Calculus II
Final Test; May 4, 2005

Time limit: 150 min.

Your name (print):

Your signature:

Each problem is worth of 10 points.

1. Evaluate the indefinite integral

$$\int \frac{\sin \frac{1}{x}}{x^2} dx.$$

2. Evaluate the definite integral

$$\int_1^9 \sqrt{t} \ln t dt.$$

3. Evaluate the indefinite integral

$$\int \cos(\sqrt{x})dx.$$

4. Evaluate the definite integral

$$\int_0^2 \frac{x-1}{x^2+3x+2}dx.$$

5. Evaluate the improper integral

$$\int_0^\infty x^2 e^{-x^3} dx.$$

6. Determine if the sequence $\{a_n\}$,

$$a_n = \frac{\ln(5 + e^n)}{5n}$$

converges or diverges. If converges, find its limit.

7. Determine if the series

$$\sum_{n=0}^{\infty} \frac{1}{n^2 + 3n + 2}$$

is convergent or divergent. If convergent, find its sum.

8. Determine if the series

$$\sum_{n=2}^{\infty} \frac{1}{n^4 - n^3}$$

is convergent or divergent.

9. Test the series

$$\sum_{n=2}^{\infty}(-1)^n \frac{\ln n}{n^2}$$

for convergence or divergence.

10. Find the Maclaurin series representation of the function

$$f(x) = \frac{1 - x^3}{1 + x^3},$$

and determine the interval of convergence.

11. Find the radius of convergence and the interval of convergence for the power series

$$\sum_{n=1}^{\infty} \frac{x^n}{n^3 3^n}.$$

12. Evaluate the indefinite integral

$$\int \frac{x}{1+x^4}dx$$

as a power series.

13. Find the Taylor series expansion of $f(x) = \ln x$ centered at $a = 3$. (You don't have to show that $R_n(x) \to 0$.)

14. Evaluate the indefinite integral

$$\int e^{-x^2} dx$$

as a power series.

15. Find the Maclaurin series of the function

$$f(x) = \frac{1}{\sqrt{1+x^2}}$$

then, by using this result, evaluate $f^{(8)}(0)$.