MA 126: Calculus II
Midterm Test #2, March 21, 2005

Time limit: 105 min.

Your name (print):

Your signature:

1. How large do we have to choose n so that the approximations T_n and M_n to the integral

$$\int_0^{\sqrt{\pi}} \sin(x^2)dx$$

are accurate within 0.0001?

10 points

2. Calculate the improper integral

$$\int_1^\infty 2^{-x} dx.$$

10 points

3. Determine if the improper integral

$$\int_0^\infty \frac{1}{(x^4+2)^{1/3}}\,dx$$

is convergent or divergent. Explain!

10 points

4. Find the number a such that the line $x = a$ bisects the area under the curve $y = \dfrac{1}{x}$, $1 \leq x \leq 1.44$. Also, find the number b such that the line $y = b$ bisects the above area.

10 points

5. Consider the region D bounded by the curves $y = x^{2/3}$, $x = 8$, $y = 0$. Find the volume of the solid S that can be obtained by revolving D about the y-axis.

10 points

6. Find the exact length of the curve $x = e^t + e^{-t}$, $y = 2t$, $0 \leq t \leq 1$.

10 points

7. Find the centroid of the lamina (with constant density) bounded by the curves $y = 1/x$, $y = 0$, $x = 1$, $x = 4$.

10 points

8. Let $a_n = \cos\left(\frac{\pi}{2n}\right)$. Check the sequence $\{a_n\}$ for monotonicity and boundedness. Is it convergent? Explain everything!

10 points

9. The sequence $\{a_n\}$ is defined recursively by $a_1 = \sqrt{6}$, $a_{n+1} = \sqrt{6 + a_n}$ for $n = 1, 2, \ldots$. Is it convergent? Explain! Find $\lim_{n\to\infty} a_n$.

10 points

10. Find the values of x for which the series

$$\sum_{n=0}^{\infty} 3^n (x+2)^{n+1}$$

converges. Find the sum of the series for those values of x.

10 points