Calculus I

FINAL EXAM

April 28th, 2005

Name: _____

- Show your work; clearly write down each step in your calculation/reasoning. *No credit* is given for a correct numerical answer without any justification.

1. *(12 pts)* Evaluate the following limits:

(a) $\lim_{x\to 2} \frac{x^2+2x-8}{x-2}$

(b) $\lim_{x\to\infty} e^x x^{-2}$

(c) $\lim_{x\to 0^+} (1-x)^{\frac{1}{2x}}$ {Hint: use the fact that $f(x) = e^{\ln f(x)}$}

2. *(10 pts)* Find the equation of the tangent line to the graph of $y = f(x) = \frac{\ln x}{x}$ at the point (1,0).

3. *(8pts)* Differentiate

$$3e^{2x+3} + \sqrt{x}\ln x$$

4. *(8pts)* Differentiate

$$\ln\left(\cos(2x)\right)$$

5. *(10pts)* At noon, two ships start moving from the same point. Ship A is sailing south at 15 mi/h and ship B is sailing east at 20 mi/h. How fast is the distance between the ships changing at 3:00 P.M.?

6. *(12pts)* Consider the function $f(x) = x^4 + 16x^3 + 72x^2$.
(a) Where is the function increasing and where is the function decreasing? Write down your answers in interval notation.

(b) What are the local maxima and minima of $f(x)$?

(c) Where is the $f(x)$ concave up and where is the $f(x)$ concave down? Write down your answers in interval notation.

(d) What are the inflection point(s) of $f(x)$? Write down your answers in the form $(x, f(x))$.

(e) Use the above information to sketch the graph of $f(x)$.

7. *(10pts)* A rectangular storage container with a **closed** top is to have a volume of 72 ft^3. The length of its base is twice the width. Find the dimensions of the box that minimize the amount of material used.

8. (a) *(5pts)* Find the most general antiderivative of the function $f(x) = e^{x+2} + 3\sec^2 x$.

(b) *(5pts)* Find $g(x)$ when $g''(x) = 6x^2 + e^x$, $g(2) = e^2$ and $g'(0) = 5$.

9. *(10pts)* Evaluate the following integral:

$$\int_0^4 3x(\sqrt{x} + x)\, dx$$

10. *(10pts)* Evaluate the following integral:

$$\int_1^{\sqrt{3}} \frac{1}{5x} + \frac{2}{1+x^2}\, dx$$

(BONUS, 5pts) Find an antiderivative for $\ln x$. {Hint: What is the derivative of $x \ln x$? }