MA 125-5B, Spring 2004

FINAL EXAM

May 3, 2004 (150 minutes)

Name: SSN:

Max. Points: 100 + 5 Bonus Points: Exam Grade:

Turn in **all the work** which you did to solve the problems, not just the final answer. In particular, include **intermediate steps in calculations**, wherever they demonstrate which method you used to get the result. You may use separate sheets for this.

The exam is **closed book** and **closed notes**. **No calculator** is to be used.

1. Find the following limits $(4 + 4 + 4$ pts):

(a) $\lim_{x\to\infty} \dfrac{x^2+1}{x-4x^2}$

(b) $\lim_{x\to 0} \dfrac{1-\cos x}{x^2}$

(c) $\lim_{x \to \infty} \dfrac{\ln x}{x}$

2. Use the definition of the derivative to find $f'(x)$ for the function $f(x) = x^3$ (5 pts).

3. Find the equation for the tangent line to the graph of the function $f(x) = \ln(x^2 - 3)$ at the point $(x, y) = (2, 0)$ (5 pts).

4. The graph of the derivative f' of a function f is provided. Use it to find the following:

(a) intervals of increase and decrease for f (2 pts),

(b) local minima and local maxima of f (2 pts),

(c) intervals where the graph of f is concave upwards or concave downwards (2 pts),

(d) inflection points of f (2 pts).

(e) Assume that $f(0) = 0$. Sketch the graph of f (4 pts).

5. Find the derivative $f'(x)$ for the following functions $(4+4+4+4$ pts):

(a) $f(x) = x \sin x$

(b) $f(x) = 3e^{3x} + e^{x^3}$

(c) $f(x) = \dfrac{x^2 - 1}{x^2 + 1}$

(d) $f(x) = \sqrt{\tan(2x)}$

6.* (a) Use implicit differentiation to find y' in terms of x and y for the curve $x^3y = y^3 + x^3$. (5 pts)

7. Find all critical numbers as well as the absolute maximum and absolute minimum of $f(x) = x^3 - 3x^2 + 1$ in the domain $-\frac{1}{2} \leq x \leq 4$. (8 pts)

8. Find the point on the line $y = -2x + 2$ which has the shortest distance from the origin. (8 pts)

9. (a) State the Mean Value Theorem. (3 pts)

(b) State the Evaluation Theorem. (3 pts)

10. Let $f(x) = \sin x$.

(a) Find the definite integral of f from $x = 0$ to $x = 2\pi$. (4 pts)

(b) Find the total area between the graph of f and the x-axis from $x = 0$ to $x = 2\pi$. (4 pts)

11. Calculate the following definite and indefinite integrals: $(4 + 4 + 4$ pts)

(a) $\displaystyle\int_1^4 \frac{x - \sqrt{x}}{\sqrt{x}}\, dx$

(b) $\displaystyle\int_0^1 10^x\, dx$

(c) $\int \left(\dfrac{1}{x^2+1} - \dfrac{2}{\sqrt{1-x^2}} \right) dx$

12. The acceleration (in m/sec^2) of a particle which moves along a straight line is given by $a(t) = 3t^2$. It is also know that its velocity (in m/sec) and position (in meters) at time $t = 0$ are $v(0) = 2$ and $s(0) = -2$. Find the position function $s(t)$ for all times t. (8 pts)